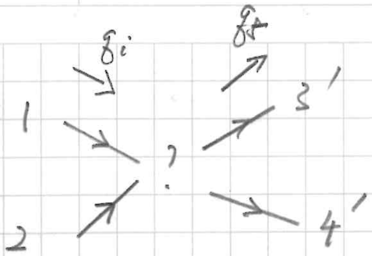
Discovery of W-boson



Non-relativistic formulation

Fermi Golden Rule

$$\frac{d\sigma}{d\Omega} = 2\pi\, |M|^2 \, \frac{1}{v_i} \, \frac{1}{(2\pi)^3} \, q_f^2 \, \frac{dq_f}{d\sqrt{s}}$$

(at given E or S.)

density of final states

$$N_f = V \int \frac{d^3 q_f}{(2\pi)^3}$$

$$\frac{d^2 N_f}{d\sqrt{s}\, d\Omega}$$

CM frame

$$\sqrt{s} = \sqrt{q_i^2 + m_1^2} + \sqrt{q_i^2 + m_2^2}$$

$$= \sqrt{q_f^2 + m_3'^2} + \sqrt{q_f^2 + m_4'^2}$$

$$v_i = \frac{d\sqrt{s}}{dq_i} \longrightarrow$$ how total E change with q_i

$$= q_i \left(\frac{1}{E_1} + \frac{1}{E_2} \right) = \frac{q_i \sqrt{s}}{E_1 E_2}$$

$$v_f = \frac{q_f \sqrt{s}}{E_3' E_4'}$$

if $m_3', m_4' \to 0$

$\sqrt{s} = 2 q_f \qquad v_f \to 2$

$c \longleftarrow \longrightarrow c$

for decay : very similar

$$\frac{d\Gamma}{d\Omega} = 2\pi |M|^2 \frac{1}{(2\pi)^3} q_f^2 \frac{dq_f}{d\sqrt{s}}$$

" ν always left handed "

$V-A$
helicity
etc.
Perkins P.152

e.g.

$$\nu_e \; e \rightarrow \nu_e \; e$$

$$\frac{d\sigma}{d\Omega} = (2\pi) |M|^2 \frac{1}{v_i} \frac{1}{(2\pi)^3} q_f^2 \frac{dq_f}{d\sqrt{s}} \; 2$$

sum over final states

av. over initial

$M_\nu \rightarrow 0$

$M_e \approx 0$

$v_i \approx 2$

$$\frac{4 G_F}{\sqrt{2}}$$

$$M \rightarrow \frac{g_W^2}{2} \frac{1}{M_W^2} \rightarrow$$ fac. of 2 is only the convention

$$\sqrt{s} \approx 2 q_f$$

$$\frac{dq_f}{d\sqrt{s}} = \frac{1}{2}$$

$$\frac{d\sigma}{d\Omega} \rightarrow 2\pi \frac{g_W^4}{4 M_W^4} \frac{1}{2} \frac{1}{8\pi^3} q_f^2 \frac{1}{2} (2)$$

$$= \frac{g_W^4}{32 M_W^4} \frac{1}{\pi^2} q_f^2$$

identity

$$\frac{1}{2}G_F^2 = \left(\frac{g_W^2}{8M_W^2}\right)^2$$

$$\frac{d\sigma}{d\Omega} = G_F^2\,\frac{q_f^2}{\pi^2}$$

$$\sigma \longrightarrow G_F^2\,\frac{1}{\pi}\left(4q_f^2\right)$$

why energetic ν is easier to detect

e.g. Homestake exp.

$$= G_F^2\,\frac{1}{\pi}\,s \qquad //$$

(nice result)

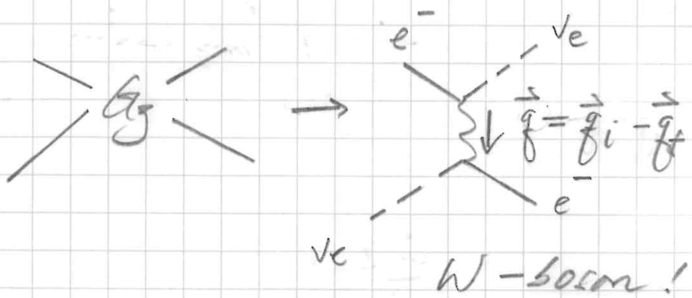
Perkins 5.24

* But

if $s \longrightarrow \infty$

$\sigma \longrightarrow \infty$ $\longrightarrow$ what's wrong?

catastrophe



W-boson!

$\Rightarrow$ Discovery must happen!

$$q^2 = \left(E_e - E_{\nu_e}'\right)^2 - \left(\vec{q}_i - \vec{q}_f\right)^2$$

$\left(E_e - E_{\nu_e}'\right)^2 \simeq 0$

$q_i = q_f$

$$\approx -2q_f^2\left(1-\cos\theta\right)$$

the correct expression of $\langle |M|^2 \rangle$

$$\langle |M|^2 \rangle \to \frac{g_W^4}{4 M_W^4} \left[\frac{-M_W^2}{q^2 - M_W^2} \right]^2$$

$$q^2 \simeq -2 q_f^2 (1 - \cos\theta)$$

ratio of σ :

$$r = \frac{\int_{-1}^{1} dz \left\{ \frac{1}{1 + \frac{2 q_f^2}{M_W^2}(1-z)} \right\}^2}{\int_{-1}^{1} dz}$$

$$= \frac{1}{2} \int_0^2 dz' \left\{ \frac{1}{1 + \frac{2 q_f^2}{M_W^2} z'} \right\}^2$$

$$= \frac{1}{\frac{4 q_f^2}{M_W^2}} \left\{ 1 - \frac{1}{1 + \frac{4 q_f^2}{M_W^2}} \right\}.$$

if $q_f \to 0$ $\quad r \to 1$

$q_f \to \infty$ $\quad r \to \frac{M_W^2}{4 q_f^2} \leftrightarrow \frac{M_W^2}{s}$

$$\sigma : \sigma_{g}^2 \frac{1}{\pi} s \to \sigma_g^2 \frac{1}{\pi} \frac{s}{1 + s/M_W^2}$$

$s \to \infty$

$\sigma \to G_F^2 \frac{1}{\pi} M_W^2$

effectively $s \to M_W^2$

No DIV at all

Relativistic formulation :

$1+2 \longrightarrow 3+4$

$\frac{d^3p_3}{(2\pi)^3} \frac{d^3p_4}{(2\pi)^3} \frac{1}{2\varepsilon_3} \frac{1}{2\varepsilon_4}$

$(2\pi)^4 \delta(E-\varepsilon_1-\varepsilon_2) \; \delta^3(\vec{P}-\varepsilon_1-\varepsilon_2)$

$$d\sigma = \frac{1}{4\sqrt{(p_1\cdot p_2)^2 - p_1^2 p_2^2}} |M|^2 \, d\rho_N$$

useful result

* $(p_1\cdot p_2)^2 - p_1^2 p_2^2$ is frame independent

if we pick CM frame

$$\left(\varepsilon_1\varepsilon_2 + \vec{q}_i^{\,2}\right)^2 - m_1^2 m_2^2$$

$$= \varepsilon_1^2\varepsilon_2^2 + q_i^4 + 2q_i^2\varepsilon_1\varepsilon_2 - m_1^2 m_2^2$$

$$= 2q_i^4 + q_i^2 \{ m_1^2 + m_2^2 + 2\varepsilon_1\varepsilon_2 \}$$

$$= q_i^2 \{ \varepsilon_1^2 + \varepsilon_2^2 + 2\varepsilon_1\varepsilon_2 \}$$

$$= q_i^2 s$$

$s = (\varepsilon_1+\varepsilon_2)^2$ for CM frame

for $\nu e \to \nu e$

$$\langle M^2 \rangle \xrightarrow{\text{ΔST}} \cancel{2\text{spin}} \times \frac{g_W^4}{M_W^4} (p_1\cdot p_2)(p_3\cdot p_4)$$

——— to be derived later

General $1+2 \to 3+4$ scattering

$$d\sigma = \frac{1}{4\sqrt{(p_1\cdot p_2)^2 - p_1^2 p_2^2}} \langle |M|^2 \rangle \; d\varphi_2$$

$$\frac{1}{4}\frac{1}{\sqrt{q_i^2 s}} \langle |M|^2 \rangle \quad \frac{q_f}{4\pi\sqrt{s}} \frac{1}{4\pi} d\Omega$$

$$= \frac{1}{64\pi^2} \frac{q_f}{q_i} \frac{1}{s} \langle |M|^2 \rangle \frac{d\Omega}{4\pi}$$

famous factor

Griffith 6.42

for $e\nu \to e\nu$

neglecting M_e also:

$\sim p_1 \cdot p_2$ $m_i \to 0$

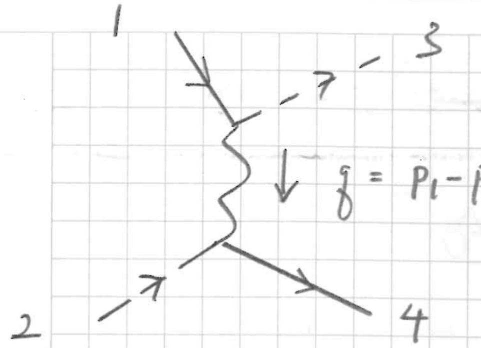
$$\langle |M|^2 \rangle \to 2_{spin} \frac{g_W^4}{M_W^4} 2q_i^2 \; 2q_f^2 \quad \& \quad q_i = q_f$$

Griffith 10.11

$$\frac{d\sigma}{d\Omega} \to \frac{g_W^4}{M_W^4} \frac{1}{32\pi^2} q_f^2$$

$$= \frac{G_F^2}{\pi^2} q_f^2 \quad //$$

Exactly as obtained



$$q = p_1 - p_3 = p_4 - p_2$$

e.g.

$e + \nu_\mu \to \mu + \nu_e$

$$\mathcal{M} = \frac{g_W^2}{8 M_W^2} \bar{u}_3 \gamma^\mu (1-\gamma_5) u_1 \; \bar{u}_4 \gamma_\mu (1-\gamma_5) u_2$$

$$\sum_{\text{spins}} |\mathcal{M}|^2 \to \frac{g_W^2}{8 M_W^2} t_1 \; t_2$$

$$t_1 = \mathrm{tr}\left[\gamma^\mu (1-\gamma_5)(\not{p}_1 + m_e)\gamma^\nu (1-\gamma_5)\not{p}_3\right]$$

$$t_2 = \mathrm{tr}\left[\gamma_\mu (1-\gamma_5)\not{p}_2 \gamma_\nu (1-\gamma_5)(\not{p}_4 + m_\mu)\right]$$

$$t_1 \to \mathrm{tr}\left(\gamma^\mu (1-\gamma_5)^2 \not{p}_1 \gamma^\nu \not{p}_3\right)$$

m_e term can't contribute

$$= 2\,\mathrm{tr}\left(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3\right) + 2\,\mathrm{tr}\left(\gamma_5 \gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3\right)$$

$$= 8 \left\{ p_1^\mu p_3^\nu + p_3^\mu p_1^\nu - (p_1 \cdot p_3) g^{\mu\nu} - i \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{3\beta} \right\}.$$

$$t_2 \to 8 \left\{ p_{2\mu} p_{4\nu} + p_{4\mu} p_{2\nu} - (p_2 \cdot p_4) g_{\mu\nu} - i \varepsilon_{\mu\nu\alpha\beta} p_2^\alpha p_4^\beta \right\}$$

$$t_1 \cdot t_2 = 64 \times \{ \ \cdots \ \}.$$

$$\left.\begin{array}{l} -2\, p_1\cdot p_2 \ \ p_2\cdot p_4 \\ -2\, p_2\cdot p_4 \ \ p_1\cdot p_3 \\ \\ 4\, p_1\cdot p_3 \ \ p_2\cdot p_4 \end{array}\right\} \ 0$$

$\longrightarrow$ $g^{\mu\nu} g^{\mu\nu}$ term.

$2\, p_1\cdot p_2 \ \ p_3\cdot p_4$

$2\, p_1\cdot p_4 \ \ p_2\cdot p_3$ $\longrightarrow$ 2 sym. term

— cancel!

$$2 \{ p_1\cdot p_2 \ \ p_3\cdot p_4 - p_1\cdot p_4 \ \ p_2\cdot p_4 \}. \longrightarrow$$

$$- \underline{\varepsilon^{\mu\nu\lambda\sigma} \varepsilon_{\mu\nu\kappa\tau}} \ p_{1\lambda} p_{3\sigma} p_2^{\kappa} p_4^{\tau}$$

$$-1 \quad -2\left(\delta^\lambda_\kappa \delta^\sigma_\tau - \delta^\lambda_\tau \delta^\sigma_\kappa\right)$$

<u>Griffith p. 303</u>

Putting it all together:

$$\langle M^2 \rangle_{AST} = \frac{1}{2} \sum_{\text{spins}} |M|^2$$

sum
over final
av.
over initial

stare at it!
beautiful result

$$\langle M^2 \rangle_{AST} = \frac{1}{2} \frac{g_W^4}{64 M_W^4} t_1 t_2 = \frac{1}{2} 4 (p_1\cdot p_2)(p_3\cdot p_4) \frac{g_W^4}{M_W^4}$$

//

writing out the index

Appendix

$$\bar{u}_{1\alpha}\, \Gamma_{\alpha\beta}\, u_{2\beta}$$

$$\mathcal{M} = \bar{u}_1 \Gamma u_2$$

$$|\mathcal{M}|^2 \to \bar{u}_1 \Gamma u_2 \left(\bar{u}_1 \Gamma u_2\right)^*$$

$$= \left(\bar{u}_1 \Gamma u_2\right)\left(u_2^\dagger \Gamma^\dagger \gamma^0 u_1\right)$$

$$= \mathrm{tr}\left[\Gamma\; u_2 \bar{u}_2\; \bar{\Gamma}\; u_1 \bar{u}_1\right]$$

$$\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$$

where

$$\left(u_1 \bar{u}_1\right)_{\alpha\beta} = \left(\not{p}_1 + m_1\right) \quad \text{etc}$$

Note also

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0 = \begin{cases} \gamma^0 \\ -\vec{\gamma} \end{cases}$$

$$\gamma_5^\dagger = \gamma_5$$

e.g. $\left|\left(\bar{u}_3 \gamma^\mu (1-\gamma_5) u_1\right)\right|^2$

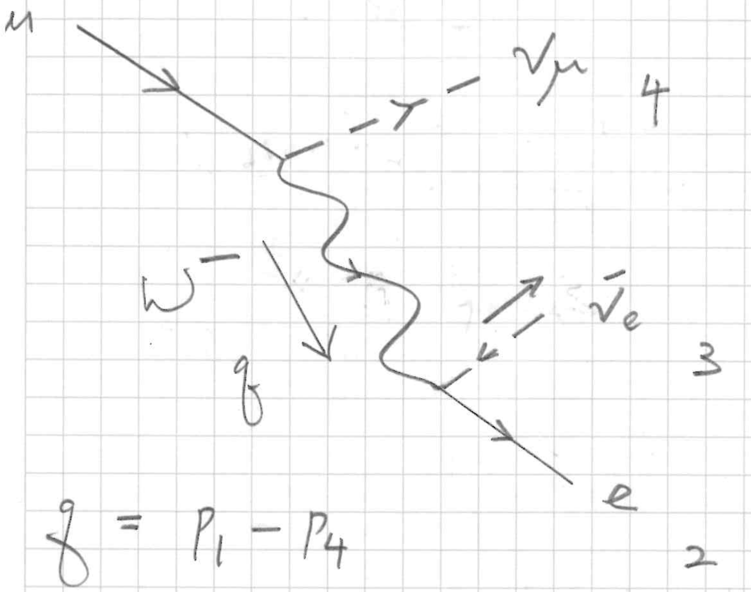
$$\to \bar{u}_3 \gamma^\mu (1-\gamma_5) u_1 \quad u_1^\dagger (1-\gamma_5) \gamma^{\nu\dagger} \gamma^0 u_3$$

$$= \mathrm{tr}\left[\gamma^\mu (1-\gamma_5) u_1 \bar{u}_1\; \gamma^\nu (1-\gamma_5)\; u_3 \bar{u}_3\right]$$

Muon Decay

$\mu \longrightarrow e \quad \bar{\nu}_e \quad \nu_\mu$

1 $\quad$ 2 $\quad$ 3 $\quad$ 4



CM-frame = frame of rest μ

$P_1 = P_2 + P_3 + P_4$

$q = P_1 - P_4$

$\cancel{\text{sign}}$ $i\mathcal{M} \sim (-ig_w)^2 \ldots \frac{-i g_{\mu\nu}}{q^2 - M_W^2}$

$\mathcal{M} \longleftrightarrow \frac{-g_w^2}{M_W^2}$

$$\mathcal{M} = \frac{-g_w^2}{8 M_W^2} \quad \bar{u}_4 \gamma^\mu (1-\gamma_5) u_1$$
$$\bar{u}_2 \gamma_\mu (1-\gamma_5) v_3$$

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{2} \frac{g_w^4}{64 M_W^4} \, t_1 \cdot t_2$$

$$t_1 = \mathrm{tr}\left[\gamma^\mu (1-\gamma_5)(\not{P}_1 + m_\mu) \gamma^\nu (1-\gamma_5) \not{P}_4 \right]$$

$$t_2 = \mathrm{tr}\left[\gamma_\mu (1-\gamma_5) \not{P}_3 \, \gamma_\nu (1-\gamma_5)(\not{P}_2 + m_e) \right]$$

t_1 : m_μ can't contribute when m_{ν_μ} is massless.

$$t_1 = 8 \{ p_1^\mu p_4^\nu + p_4^\mu p_1^\nu - g^{\mu\nu}(p_1 \cdot p_4)$$
$$- i \varepsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{4\beta} \}$$

$$t_2 = 8 \{ p_{3\mu} p_{2\nu} + p_{2\mu} p_{3\nu} - g_{\mu\nu}(p_2 \cdot p_3)$$
$$- i \varepsilon_{\mu\nu\alpha\beta} p_3^\alpha p_2^\beta \}.$$

$$t_1 \cdot t_2 = 64 \times \{\cdots\}$$

$$\{\cdots\} = 4 (p_1 \cdot p_3)(p_4 \cdot p_2).$$

$p_1 : [m_\mu , \vec{0}]$ $\qquad p_1 = p_2 + p_3 + p_4.$

$$p_1 \cdot p_3 = m_\mu E_3$$
$$p_2 \cdot p_4 = \frac{1}{2}\left[(p_2 + p_4)^2 - p_2^2 - p_4^2\right]$$
$$= \frac{1}{2}\left[(p_1 - p_3)^2 - m_2^2 - m_4^2\right]$$
$$\frac{1}{2}\left(m_\mu^2 - 2 m_\mu E_3 + \cancel{m_3^2} - m_2^2 - \cancel{m_4^2}\right)$$

$$\langle |M|^2 \rangle = \frac{g_W^4}{M_W^4} m_\mu E_{\bar{\nu}_e} \left(m_\mu^2 - m_e^2 - 2 m_\mu E_{\bar{\nu}_e}\right) //$$

$$d\Gamma_{\mu \to e \bar{\nu}_e \nu_\mu} = \frac{1}{2 m_\mu} \langle |M|^2 \rangle \underline{d\Phi_3}$$

$$\frac{d^3p_2 \, d^3p_3 \, d^3p_4}{\left[(2\pi)^3\right]^3} \frac{1}{2E_2 \, 2E_3 \, 2E_4}$$

$$(2\pi)^4 \delta^4 \begin{matrix} m_\mu - E_2 - E_3 - E_4 \\ \vec{0} - \vec{p}_2 - \vec{p}_3 - \vec{p}_4 \end{matrix}$$

$$\frac{1}{2 m_\mu} \times \frac{g_W^4}{M_W^4} \times$$

can be neglected

can't be neglected

$$\int d\Phi_3 \quad \underline{m_\mu E_{\bar{\nu}_e} \left\{ m_\mu^2 - m_e^2 - 2 m_\mu E_{\bar{\nu}_e} \right\}}$$

see calculation notes.

$$\Downarrow$$

$$\Gamma = \frac{g_W^4}{M_W^4} \frac{m_\mu^5}{24 \, (256) \pi^3} \longleftrightarrow \frac{G_F^2 \, m_\mu^5}{192 \pi^3} \; //$$

$$\frac{1}{2} G_F^2 = \frac{g_W^4}{64 M_W^4}$$

Dalitz Decay

see Num QM for actual calculations

$\mu \to e \, \bar{\nu}_e \, \nu_\mu$

$n \to p \, \bar{\nu}_e \, e$ $\Rightarrow$ it is essential to take m_e into account

relevant scale $\Rightarrow$ $m_n - m_p$; m_e

$\Rightarrow$ $\forall$ important

lifetime:

μ : 2×10^{-6} s

n : 12 min. $\Rightarrow$ $\forall$ because of the kinematics $m_e \neq 0$

with this naive calculation

$\Rightarrow$ 1340 s $\rightarrow$ overestimated

n, p are not pointlike

$1 - \gamma_5 \rightarrow [c_V - c_A \gamma_5]$

$c_V \approx 1$, $c_A \approx 1.26$

$\Leftarrow$

↑ width

↓ lifetime

technical notes #1 $1 \to 2, 3, 4$

$$\int d\Phi_3 = \int \frac{1}{32\pi^3}\, dE_2\, dE_3$$ invar. mass form

$$= \int \frac{1}{32\pi^3}\frac{1}{4s}\, ds_{34}\, ds_{24}$$ energy form

why?

$$s_{34} = (p_3+p_4)^2 = (P - p_2)^2 = s + m_2^2 - 2\sqrt{s}\,E_2$$

$$P = p_1 = (m_1, \vec{0})$$

$$\Rightarrow \quad ds_{34} = -2\, m_1\, dE_2$$

similarly $\quad ds_{24} = -2\, m_1\, dE_3$

The challenging task is to determine the range of $\{s_{34}, s_{24}\}$ or $\{E_2, E_4\}$;

$\{s_{34}, s_{24}\}$

① take s_{34} to be free (integrated last)

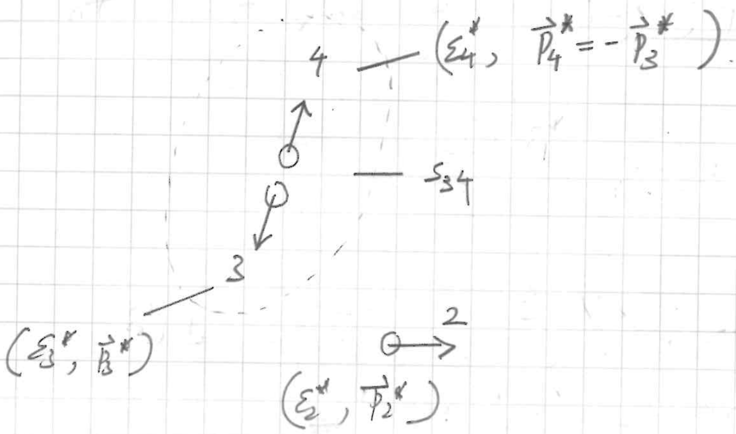
s_{34} : $(m_3+m_4)^2 \to (\sqrt{s}-m_2)^2$ or $0 \to s$ for massless case

② Given $s_{34} \longrightarrow$ determine the range of s_{24}

$\Rightarrow$ Go to the rest frame of $(3,4)$

$*$-frame.

rest frame of (3,4)



$$s_{24} = (p_2 + p_4)^2 = m_2^2 + m_4^2 + 2\left(\varepsilon_2^* \varepsilon_4^* - p_2^* p_4^* z_{24}^*\right)$$

$$s_{24}\,{}^{max}_{min} = m_2^2 + m_4^2 + 2\left(\varepsilon_2^* \varepsilon_4^* \pm p_2^* p_4^*\right)$$

$\Rightarrow$ we need $\varepsilon_2^*, \varepsilon_4^*$ (p_2^*, p_4^* can then be fixed)
@ fixed s_{34}

idea: rest frame is a good projector $\rightarrow$ only 0-th component

ε_4^*:

$$p_{34} \cdot p_4 = \sqrt{s_{34}}\, \varepsilon_4^* = (p_3 + p_4) \cdot p_4$$

$$= -\frac{1}{2}\left\{ p_3^2 - p_4^2 - p_{34}^2 \right\}$$

$$= \frac{1}{2}\left(s_{34} + m_4^2 - m_3^2 \right)$$

$$\varepsilon_4^* = \frac{1}{2\sqrt{s_{34}}}\left(s_{34} + m_4^2 - m_3^2 \right) \quad //$$

ε_2^*

$$p_{34} \cdot p_2 = \sqrt{s_{34}}\, \varepsilon_2^* = (p_3 + p_4) \cdot p_2$$

$$= \frac{1}{2}\left(s - s_{34} - m_2^2 \right)$$

$$\varepsilon_2^* = \frac{1}{2\sqrt{s_{34}}}\left(s - s_{34} - m_2^2 \right) \quad //$$

thus $\varepsilon_2^*, \varepsilon_4^* \rightarrow p_2^*, p_4^*$ are determined

$$\int d\Phi_3 \cdots \rightarrow \int ds_{34} \int ds_{24} \frac{1}{32\pi^3} \frac{1}{4s} \cdots$$

$s_{34} : (m_3+m_4)^2 \rightarrow (\sqrt{s}-m_2)^2$

$s_{24} : m_2^2+m_4^2+2(\varepsilon_2^*\varepsilon_4^*-p_2^*p_4^*) \rightarrow m_2^2+m_4^2+2(\varepsilon_2^*\varepsilon_4^*+p_2^*p_4^*)$

massless case :

$s_{34} : 0 \rightarrow s$

$s_{24} : 0 \rightarrow s - s_{34}$

$$\int d\Phi_3 \longrightarrow \frac{1}{32\pi^3} \frac{1}{4s} \int_0^s ds_{34} \int_0^{s-s_{34}} ds_{24}$$

$$= \frac{s}{256\pi^3} \quad //$$

Also we can go to the energy space

$$\varepsilon_2 = \frac{s+m_2^2-s_{34}}{2\sqrt{s}} \qquad \varepsilon_3 = \frac{s+m_3^2-s_{24}}{2\sqrt{s}}$$

for massless case

$\varepsilon_2 : 0 \rightarrow \frac{\sqrt{s}}{2}$ $\qquad \varepsilon_3^{min} \rightarrow \frac{s_{34}}{2\sqrt{s}} = \frac{\sqrt{s}}{2} - \varepsilon_2$

$\qquad \varepsilon_3^{max} \rightarrow \frac{\sqrt{s}}{2}$

$$\int d\Phi_3 \rightarrow \frac{1}{32\pi^3} \int_0^{\sqrt{s}/2} d\varepsilon_2 \int_{\sqrt{s}/2-\varepsilon_2}^{\sqrt{s}/2} d\varepsilon_3 = \frac{1}{256\pi^3} s \quad //$$

$\{E_2, E_3\}$ space is useful

don't neglect → fac → 1/2

$$\Gamma_1 = \int_0^{m_1/2} dE_2 \int_{\frac{1}{2}m_1 - E_2}^{m_1/2} dE_3 \quad \frac{1}{2m_1} \frac{g_W^4}{M_W^4} \times m_1 E_3 (m_1^2 - 2m_1 E_3) \frac{1}{32\pi^3}$$

$$= \frac{1}{64\pi^3} \frac{g_W^4}{M_W^4} m_1^5 \times \underline{\int_0^{1/2} dx_2 \int_{\frac{1}{2}-x_2}^{1/2} dx_3 \quad x_3 (1 - 2x_3)}$$

where $x_{2,3} = \dfrac{E_{2,3}}{m_1}$

$$\rightarrow \frac{1}{64\pi^3} \frac{g_W^4}{M_W^4} m_1^5 \times \int_0^{1/2} dx_2 \underline{\frac{1}{2} x_2^2 \left(1 - \frac{4}{3} x_2\right)}$$

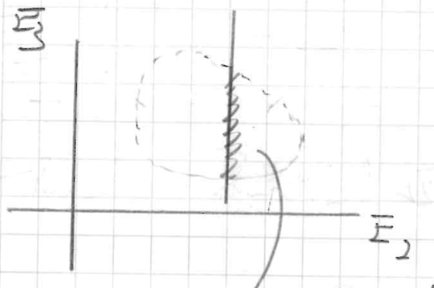
↳ 1/96.

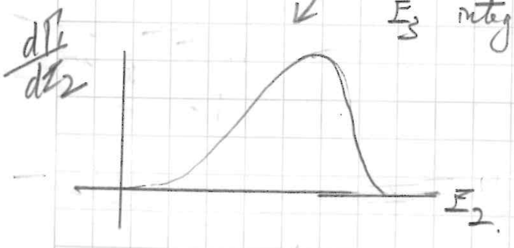
$$= \frac{m_1^5}{6144\pi^3} \frac{g_W^4}{M_W^4} \quad //$$

<u>note</u>

$$\frac{d\Gamma_1}{dE_2} = \frac{1}{128\pi^3} \frac{g_W^4}{M_W^4} m_1^4 \left(\frac{E_2}{m_1}\right)^2 \times \left(1 - \frac{4}{3}\frac{E_2}{m_1}\right)$$

if we don't finish the last integral...





technical notes #2

fac. in σ :

$$(p_1 \cdot p_2)^2 - p_1^2 p_2^2$$

$$\left\{ \frac{1}{2}\left[(p_1+p_2)^2 - p_1^2 - p_2^2 \right] \right\}^2 - m_1^2 m_2^2$$

$$\frac{1}{4}\left\{ s - m_1^2 - m_2^2 \right\}^2 - m_1^2 m_2^2$$

$$= \frac{1}{4}\left\{ s - m_1^2 - m_2^2 + 2m_1 m_2 \right\}$$

$$\left\{ s - m_1^2 - m_2^2 - 2m_1 m_2 \right\}.$$

$$= \frac{1}{4}\left[s - (m_1+m_2)^2 \right]\left[s - (m_1-m_2)^2 \right]$$

$$= s\, q^2$$

where $q = \frac{1}{2}\sqrt{s}\sqrt{1 - \frac{(m_1+m_2)^2}{s}} \times \sqrt{1 - \frac{(m_1-m_2)^2}{s}}$

also in CM frame:

$$(p_1 \cdot p_2)^2 - p_1^2 p_2^2$$

$$= (\varepsilon_1 \varepsilon_2 + q^2)^2 - m_1^2 m_2^2$$

$$= \varepsilon_1^2 \varepsilon_2^2 + 2\varepsilon_1 \varepsilon_2 q^2 + q^4 - m_1^2 m_2^2$$

$$= 2q^4 + q^2 \{ m_1^2 + m_2^2 + 2\varepsilon_1 \varepsilon_2 \}$$

$$= q^2 \{ \varepsilon_1^2 + \varepsilon_2^2 + 2\varepsilon_1 \varepsilon_2 \}$$

$$= q^2 (\varepsilon_1 + \varepsilon_2)^2 \longrightarrow q^2 s$$

this result is frame independent

at the same time proving

$$q = \frac{1}{2}\sqrt{s}\sqrt{1 - \frac{(m_1+m_2)^2}{s}}\sqrt{1 - \frac{(m_1-m_2)^2}{s}}$$