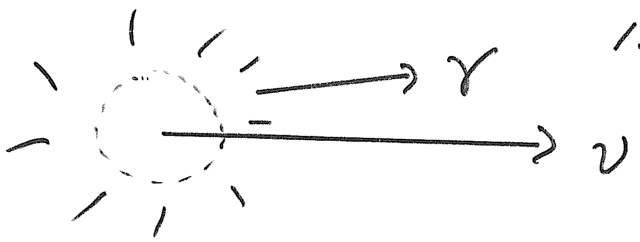
Hadrons class

11. 2025

(1)

lessons from Fermi's theory

Solar neutrinos



1.4 eV

MeV

very different interaction mechanism at work

$\Rightarrow$ weak interaction, fusion, ...

Weapons

$\frac{1}{3}$ of expected

$\longrightarrow m_\nu \neq 0 \longleftarrow$ solar ν problem

~ 10 MeV detection

$\sigma_\nu \uparrow$ as $E \uparrow$

$$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$$

$$\nu_e + n \rightarrow p + e^-$$

ν is <u>weakly</u> interacting!

< see GSI Gabriel's notes. >.

pp chain.

max. 0.42 MeV

$pp \rightarrow d + e^+ + \nu_e$

$d + p \rightarrow {}^3He + \gamma$

most ${}^4He$ is formed this way

${}^3He + {}^3He \rightarrow {}^4He + 2p.$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma.$

mostly

$e^- + {}^7Be \rightarrow {}^7Li + \nu_e$

2 Energies

10%

0.38 MeV

0.86 MeV

90%

${}^7Li + p \rightarrow 2\,{}^4He$

Rare

$p + {}^7Be \rightarrow {}^8B + \gamma$

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

$\sim$ 10 MeV detected!

${}^8Be^* \rightarrow 2\,{}^4He$

${}^{pp}\nu_e \rightarrow$ spectrum : 3-body.

${}^{7Be}\nu_e \rightarrow$ 2 lines

${}^{8B}\nu_e \rightarrow$ spectrum

$\nu_e + n \to p + e^-$ Homestake detection

$\updownarrow$

$n \to p + e^- + \bar{\nu}_e$ β-decay.

$\bar{\nu}_e + p \to n + e^+$ would be very direct

SuperKamiokande

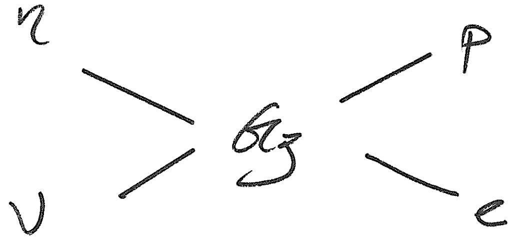
$\nu + e^- \to \nu + e^-$

$\Downarrow$

theory ? Fermi's theory

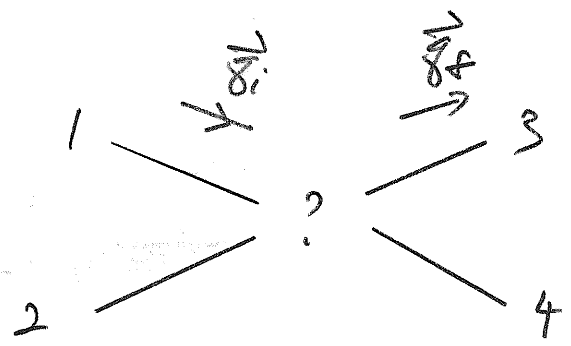
4-fermion theory

$$\mathcal{L} = -\frac{1}{\sqrt{2}} G_F \; \bar{p} \Gamma^{\mu} n \; \bar{e} \Gamma_{\mu} \nu$$



$n \to p + e^- + \bar{\nu}$

G_F : 1.166×10^{-5} GeV^{-2}



$$\frac{d\sigma}{d\Omega} = 2\pi |M|^2 \frac{1}{v_i} \frac{1}{(2\pi)^3} q_f^2 \frac{dq_f}{d\sqrt{s}}$$

at a given E or $\sqrt{s}$

density of final states

$$N_f = V \int \frac{d^3 q_f}{(2\pi)^3}$$

$$\frac{d^2 N_f}{d\sqrt{s}\, d\Omega}$$

CM frame

$$\sqrt{s} = \sqrt{q_i^2 + m_1^2} + \sqrt{q_i^2 + m_2^2}$$
$$= \sqrt{q_f^2 + m_3^2} + \sqrt{q_f^2 + m_4^2}$$

$$v_i := \frac{d\sqrt{s}}{dq_i} = \frac{q_i}{E_1} + \frac{q_i}{E_2}$$
$$= \frac{q_i \sqrt{s}}{E_1 E_2}$$

similarly

$$v_f = \frac{q_f \sqrt{s}}{E_3 E_4}$$

if $m_3\ m_4 \to 0$

$$c \longleftrightarrow c$$

$$\sqrt{s} \to 2q_f \Rightarrow v_f = 2$$

Nonsense but give the correct ans

Full sol needs relativistic formulation.

The Decay formula is similar.

$$\frac{d\Gamma}{d\Omega} = 2\pi |M|^2 \frac{1}{(2\pi)^3} q_f^2 \frac{dq_f}{d\sqrt{s}}.$$

Demonstration :

Perkins
P. 152

$\nu_e \; e \rightarrow \nu_e \; e.$

$$\frac{d\sigma}{d\Omega} = 2\pi |M|^2 \frac{1}{v_i} \frac{1}{(2\pi)^3} q_f^2 \frac{dq_f}{d\sqrt{s}} \; 2$$

sum over final states
av over initial states for e^-

$m_\nu \rightarrow 0$

$m_e \simeq 0 \qquad v_i \simeq 2$

$$M \longrightarrow \frac{1}{2} \frac{g_W^2}{M_W^2} = \frac{4G_F}{\sqrt{2}}$$

$$\sqrt{s} = 2 q_f$$

$$\frac{dq_f}{d\sqrt{s}} \longrightarrow \frac{1}{2}$$

ν : have to be left-handed due to V−A vertex

$$\frac{d\sigma}{d\Omega} = 2\pi \frac{g_W^4}{4M_W^4} \frac{1}{2} \frac{1}{8\pi^3} q_f^2 \frac{1}{2} (2)$$

$$= \frac{g_W^4}{32 M_W^4} \frac{1}{\pi^2} q_f^2 = G_F^2 \frac{q_f^2}{\pi^2}$$

$$\frac{d\sigma}{d\Omega} = G_F^2 \frac{g_f^2}{q^2}$$

S

$$\sigma \longrightarrow G_F^2 \frac{1}{q} (4 g_f^2)$$

$$= G_F^2 \frac{1}{q} S$$

$\longrightarrow$ Energetic ν is easier to detect

$//$

Feynman's WECIB (Dim. analysis)

full QFT cal gives the same result

① $\sigma \sim E^{-2}$

② $\sigma \propto |M|^2 \to G_F^2 \sim E^{-4}$

③ all masses $\longrightarrow 0$

$\Longrightarrow$ S is the only scale!

$$\sigma \to G_F^2 S$$

the only thing we don't know is $1/q$

$$\sigma = G_F^2 \frac{1}{q} S.$$

$\longrightarrow$ what happens if $S \to \infty$?

UV catastrophe!

$$\sigma \sim G_F^2 \frac{s}{\pi} \longrightarrow \infty \quad \text{as} \quad s \longrightarrow \infty$$

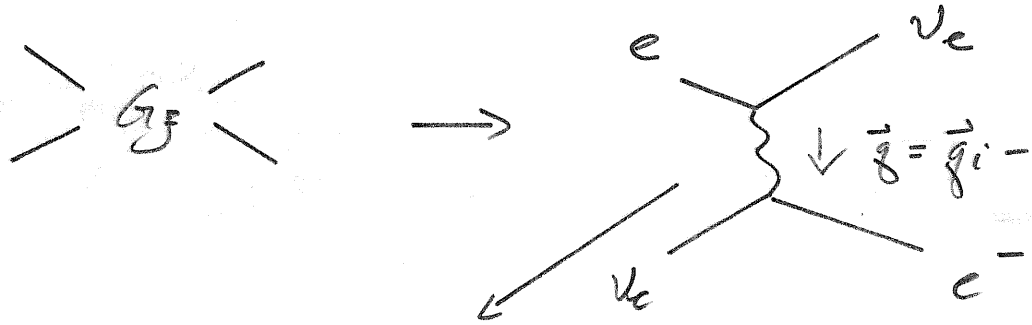
$\Longrightarrow$ signal for new physics

W-bosons

in fact

$$\sigma \longrightarrow G_F^2 \frac{M_W^2}{\pi} \quad \text{as} \quad s \longrightarrow \infty$$

with a correct calculation:



W-boson!

Discovery has to happen!

$$q^2 = (E_e - E_{\nu_e})^2 - (\vec{q}_i - \vec{q}_f)^2$$

$\nearrow$ 0 for $m_e \to 0$

$$|\vec{q}_i| = |\vec{q}_f|$$

$$\Longrightarrow -2\, q_f^2 (1 - \cos\theta)$$

$$-\frac{g_W^2}{M_W^2} \rightarrow \frac{g_W^2}{q^2 - M_W^2}$$

$$= \frac{-g_W^2}{M_W^2}\left(\frac{M_W^2}{-q^2+M_W^2}\right)$$

$$\rightarrow -\frac{g_W^2}{M_W^2}\;\underline{\frac{1}{1+\frac{2q_f^2}{M_W^2}(1-\cos\theta)}}$$

Just this factor in M.

$\Rightarrow$ ratio of σ :

$$r = \frac{\int_{-1}^{1} dz \left[\frac{1}{1+\frac{2q_f^2}{M_W^2}(1-z)}\right]^2}{\int_{-1}^{1} dz \quad 1}$$

$$= \frac{1}{2}\int_0^2 dz' \left(\frac{1}{1+\frac{2q_f^2}{M_W^2}z'}\right)^2$$

$$= \frac{1}{1+\frac{4q_f^2}{M_W^2}}$$

$q_f \rightarrow 0 \qquad r \rightarrow 1 \qquad$ old result

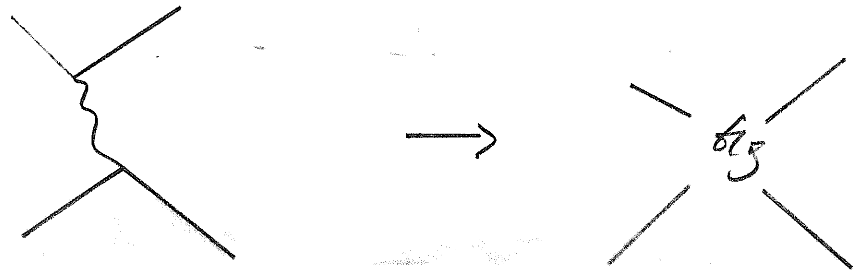
$q_f \rightarrow \infty \qquad r \rightarrow \dfrac{1}{\frac{4q_f^2}{M_W^2}} = \dfrac{M_W^2}{s}.$

$$\sigma : \frac{G_F^2}{\pi} s \xrightarrow{s \to \infty} \frac{G_F^2}{\pi} M_W^2$$

$s \mapsto M_W^2$ as promised

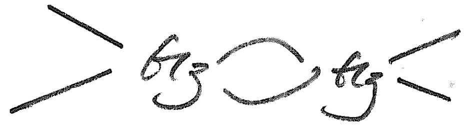
No div at all
world is saved ;)

Effective field theory:



coupling $\leftrightarrow$
integrate out heavy fields

4-fermion interaction is not renormalizable



$+ \dots$

$$\tilde{G}_F = G_F \left[1 + G_F \,\text{loop} + G_F^2 \,\text{loop}^2 + \dots \right]$$

$\text{loop} \sim \Lambda^2$ by dim analysis

but it means more UV div
as we calculate more corrections

$\longrightarrow$ failure of EFT $\longrightarrow$ a good thing: discovery

if we don't know the full theory

integrate out the heavy scales
low E theorem
couplings, etc. $\rightarrow$ χpt BSM NSL

for weak physics full EFT is known

what kind of terms are allowed in the Lagrangian? $\rightarrow$ symmetry dictates a lot

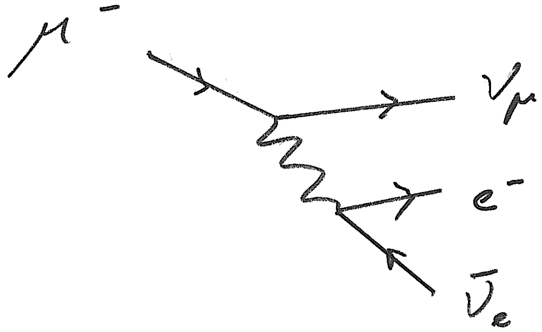
$\bar{\psi}\gamma^\mu(1-\gamma_5)\psi$ Dirac structure

V − A

$\psi \rightarrow$ spinors.

Gauge symmetries: local
global $\rightarrow$ SB.

μ decay :



full QFT calc. (study notes)

$$d\Gamma_{\mu \to e \bar{\nu}_e \nu_\mu} = \frac{1}{2 m_\mu} \langle |M|^2 \rangle \, d\Phi_3 .$$

$$\longrightarrow \frac{G_F^2 \, m_\mu^5}{192 \, \pi^3}$$

Again dim analysis gets us very far

$n \to p + \bar{\nu}_e + e$? can't take $m_e \to 0$

the scale is $m_n - m_p$; m_e is NOT small compared to that

μ : 2×10^{-6} s
n : 12 min.
} just kinematics

↳ get 1340 s in QFT calc.

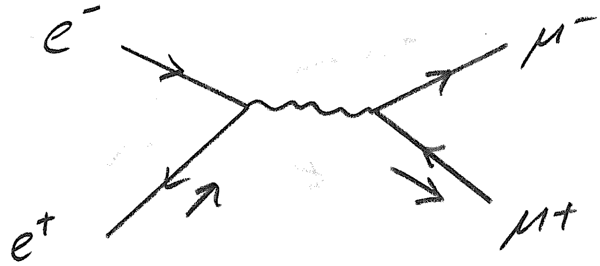
width ↑ lifetime ↓ ⟸ ↳ $1-\gamma_5$: $c_V - c_A \gamma_5$ n, p are not pointlike

$c_V \sim 1$, c_A ↳ 1.26

Counting Quarks :

(most) important QCD process

$e^+e^- \to \mu^+\mu^-$

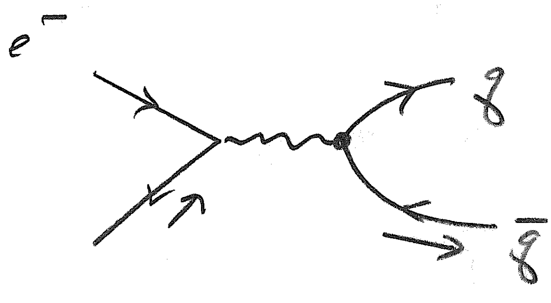


$$\sigma = \frac{4\pi}{3}\frac{\alpha^2}{s}$$

dim. analysis
(for large s)

$$\sigma_{e^+e^- \to f\bar{f}} \longrightarrow \frac{4\pi}{3}\frac{\alpha^2}{s}.$$

how to treat $u\bar{u}$ $d\bar{d}$?
$s\bar{s}$ $c\bar{c}$ ...



$$\sigma_{e^+e^- \to q\bar{q}} = Q^2 N_c \frac{4\pi}{3}\frac{\alpha^2}{s}$$

$$\longrightarrow \sum_i Q_i^2 N_c \frac{4\pi}{3}\frac{\alpha^2}{s}$$

$\longrightarrow$ count flavor w/ $\sqrt{s} > 2m_i$

$$R = \frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$$

$$= 3 \times \sum_i Q_i^2 \; \theta(\sqrt{s} - 2m_i)$$

for $\sqrt{s} >> 1$ GeV

u, d, s are excited

$$R_3 \to 3 \times \left\{ \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right\}$$

$$\simeq 2$$

if c is excited $\quad m_c \sim 1.28$ GeV

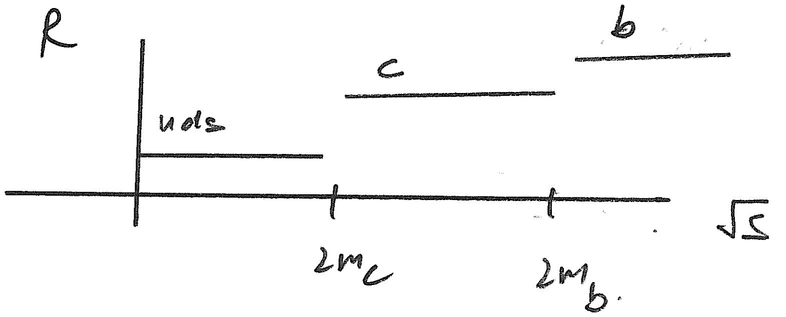
$$R_4 \sim 2 + 3 \times \left(\frac{2}{3}\right)^2$$

$$\sim 3.33$$

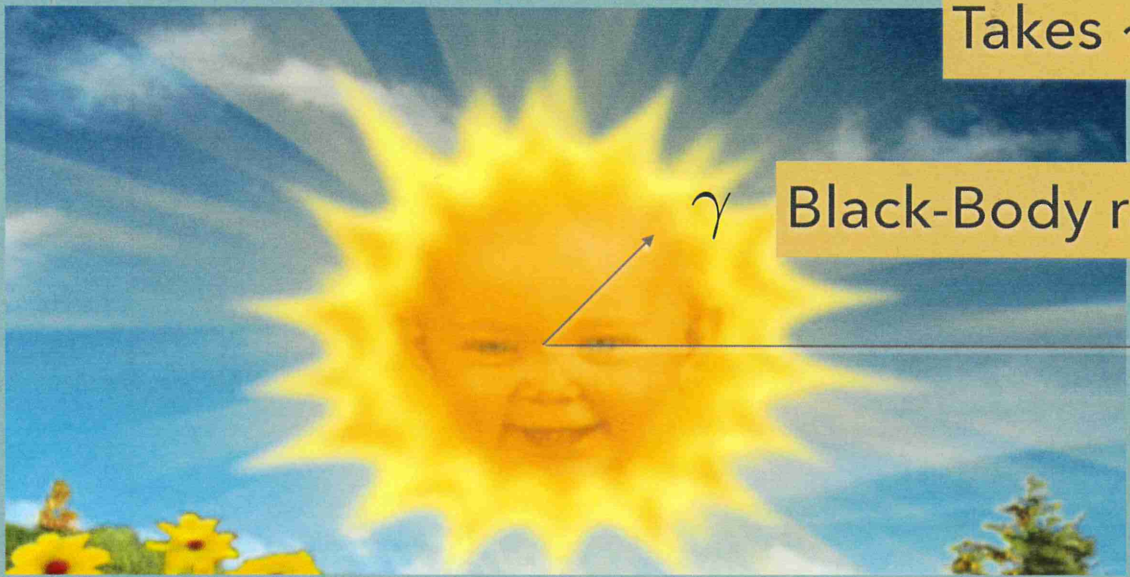
b : $\sqrt{s} > 8.4$ GeV

$R \sim 3.66$

Amazing physics!



~ Hall effect!



Takes ~500 s to reach the Earth

Black-Body radiation: 5778 K (1.4 eV)

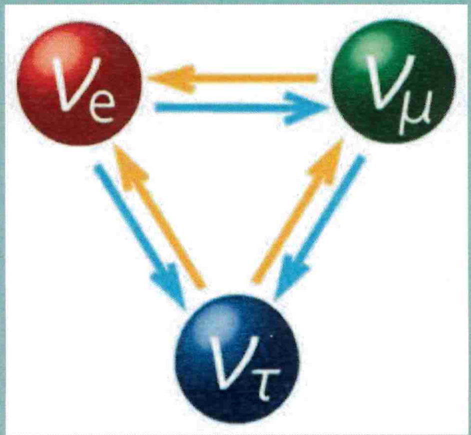
Energy scale: MeV

Homestake

$$\nu_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar}^+ + e^- .$$

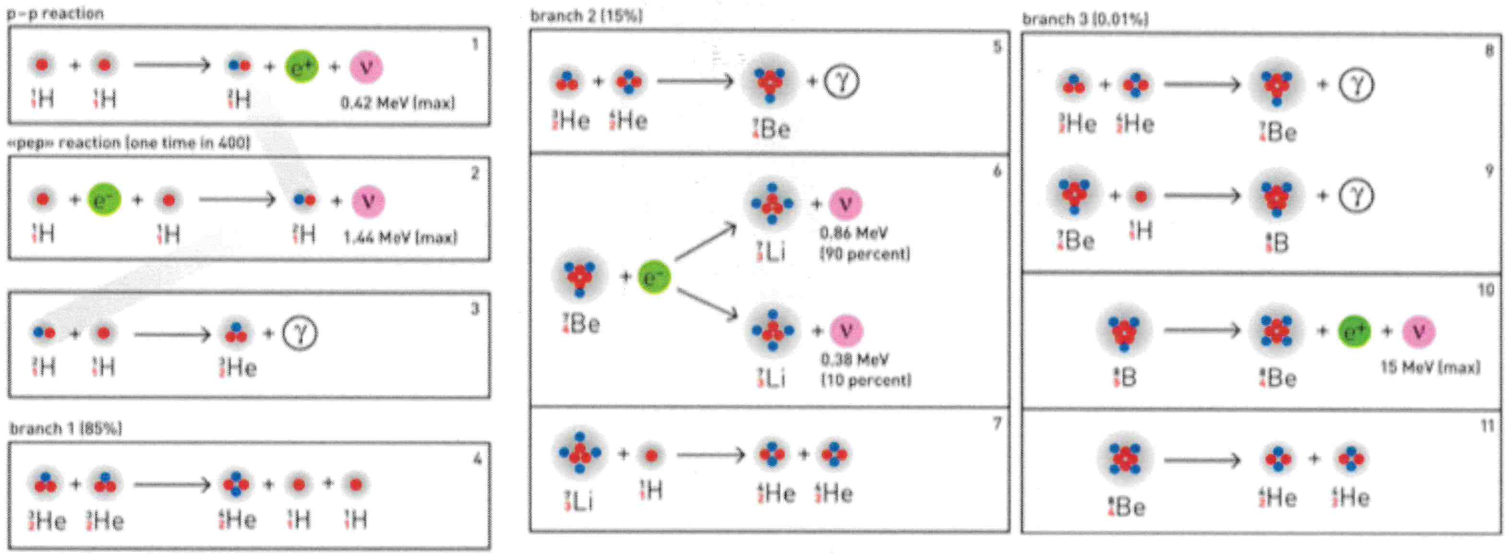
SuperKamiokande

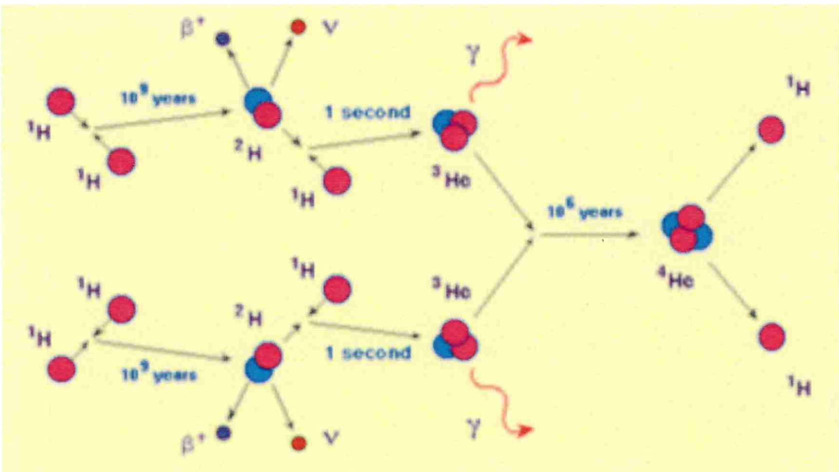
$$\nu + e^- \rightarrow \nu + e^-$$

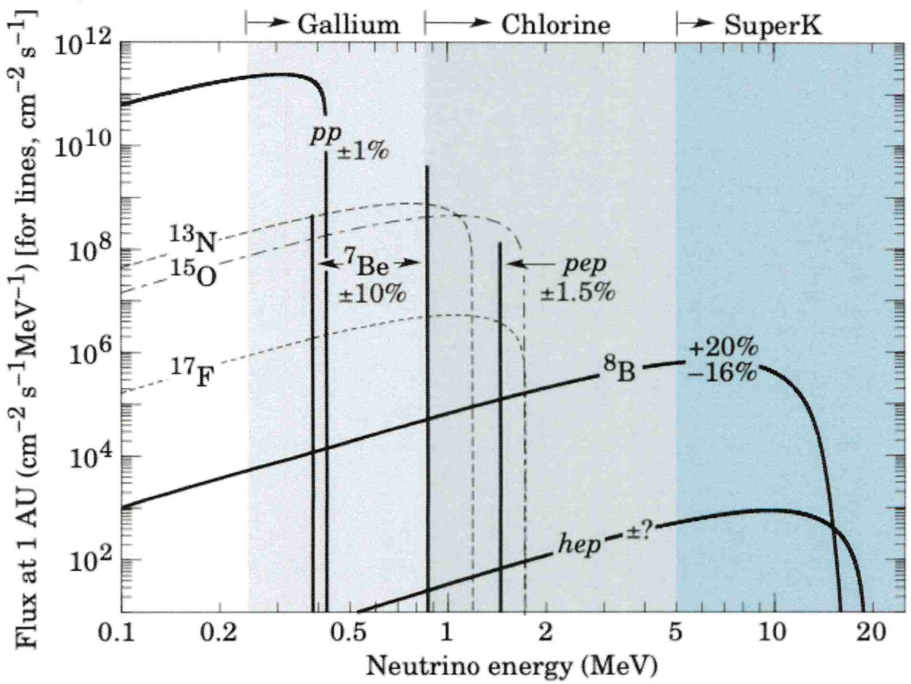


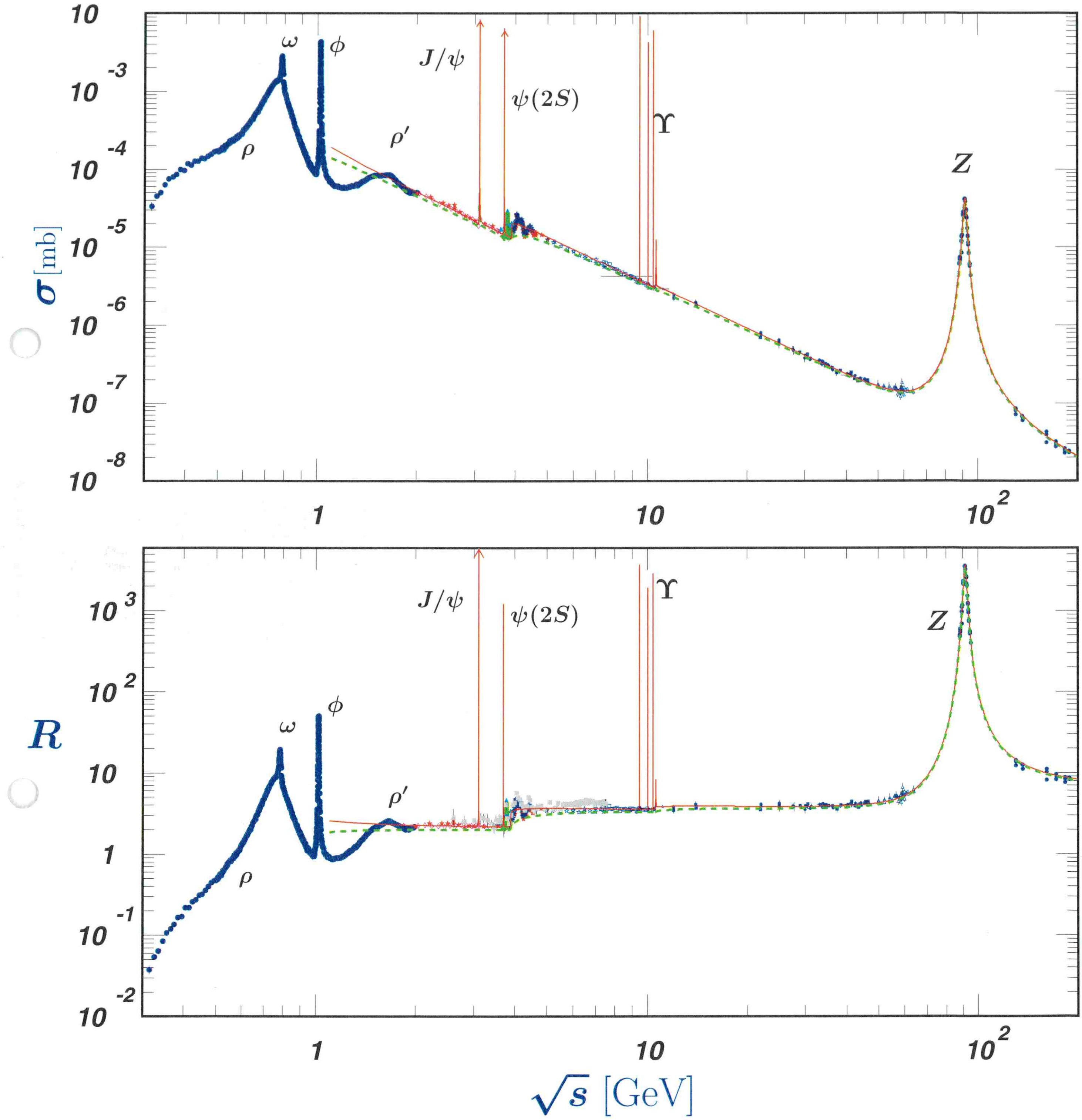
pp chains

Once ^{4}He is produced can act as catalyst initializing the ppII and ppIII chains.









σ [mb]
ω
φ
ρ
ρ′
J/ψ
ψ(2S)
Υ
Z
R
√s [GeV]