Natural Units

15/10 2022

秋

$\hbar c = 1 = 0.197\ \text{GeV fm}$

$\downarrow \qquad \hookrightarrow 10^{-15}\ \text{m}.$

$10^9\ \text{eV}$

$k_B = 1.38065 \times 10^{-23}\ \text{J K}^{-1}$

$\sim 8.6173324 \times 10^{-5}\ \text{eV K}^{-1}$

Energy $\longleftrightarrow$ Wavelength

$$E = \hbar k = \hbar \frac{2\pi}{\lambda}$$

e.g. $0.2\ \text{eV}$

$$\leftrightarrow \lambda = \frac{2\pi}{0.2\ \text{eV}}$$

$$= \frac{2\pi}{0.2}\ 0.197\ 10^9\ 10^{-15}\ \text{m}$$

$$= 6188.94\ \text{nm} \ //$$

$\sim$ infrared

Wien displacement law.

$$f_k \propto k^2 \frac{k}{e^{\beta k} - 1}$$

$$f_k' = 0 \Rightarrow 3k^2 \frac{1}{e^{\beta k} - 1} - \frac{k^3 \beta e^{\beta k}}{(e^{\beta k} - 1)^2} = 0$$

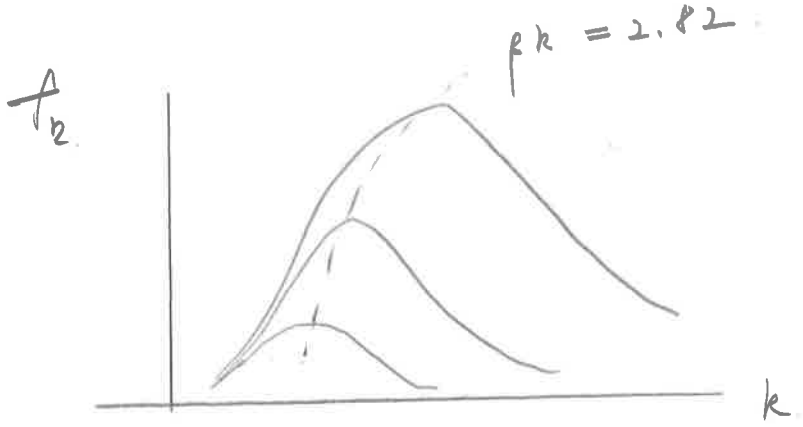
$$3(e^{\beta k} - 1) - \beta k e^{\beta k} = 0$$

$$\beta k \approx 2.8214$$

$$T_{sun} \approx 5778\ K$$

$$\varepsilon_{sun} = 2.8214 \times k_B T$$

$$= 1.405\ eV$$

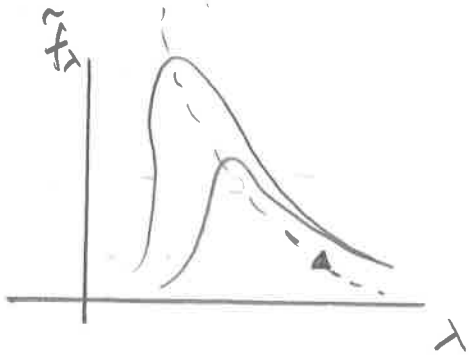


the corresponding spectrum in λ

$\beta \frac{2\pi}{\lambda} = 4.97$

$$\tilde{f}_\lambda \propto \frac{1}{\lambda^2}\,\frac{1}{\lambda^2}\,\frac{1/\lambda}{e^{\beta\frac{2\pi}{\lambda}}-1}$$

↑ photon gas factor



$$\tilde{f}_\lambda' = 0 \Rightarrow -5\,\frac{1}{\lambda^6}\,\frac{1}{e^{\beta\frac{2\pi}{\lambda}}-1} + \frac{1}{\lambda^5}\,\frac{e^{\beta\frac{2\pi}{\lambda}}\,\beta\frac{2\pi}{\lambda^2}}{\left(e^{\beta\frac{2\pi}{\lambda}}-1\right)^2} = 0$$

$$-5\left(e^{\beta\frac{2\pi}{\lambda}}-1\right) + \left(\beta\frac{2\pi}{\lambda}\right) e^{\beta\frac{2\pi}{\lambda}} = 0$$

$$\beta\frac{2\pi}{\lambda} \approx 4.9651$$

$$\lambda_{sun.} = \frac{2\pi}{4.9651}\,\frac{1}{k_B T}$$

$$\approx 2.542\ eV^{-1}$$

$$\sim 0.5 \times 10^{-6}\ m. \quad //$$

note

$$\varepsilon_{sun} \neq \frac{2\pi}{\lambda_{sun.}}$$

$\sim \left\langle \frac{1}{r} \right\rangle \neq \frac{1}{\langle r \rangle}$ but not too bad. for order of magnitude

CMB

$T \sim 2.7\ K$

$\mathcal{E}_{CMB} = 2.8214 \times k_B T$

$\rightarrow 0.656 \times 10^{-3}\ eV$

$f \sim 158.7 \times 10^9\ Hz$

$\lambda \sim 2\ mm.$ ~ microwave