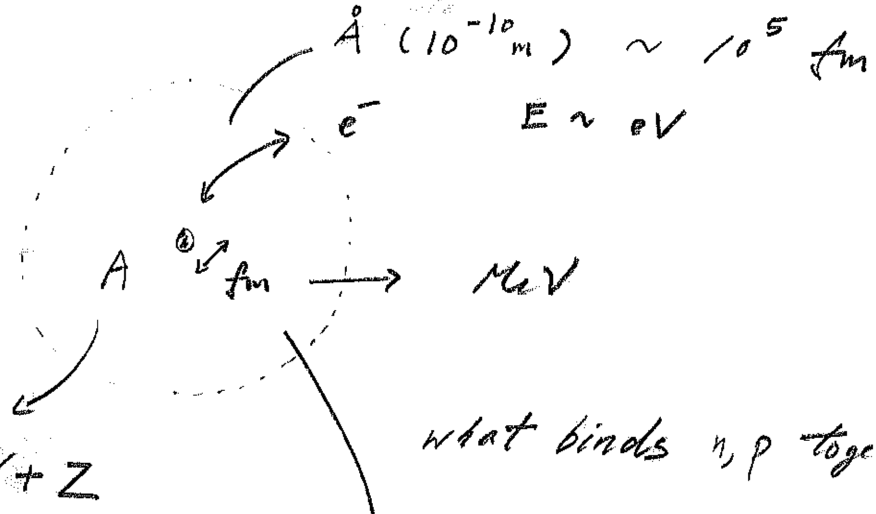
11. 2025

Introduction to Nuclear Physics

①

Atomic Nucleus

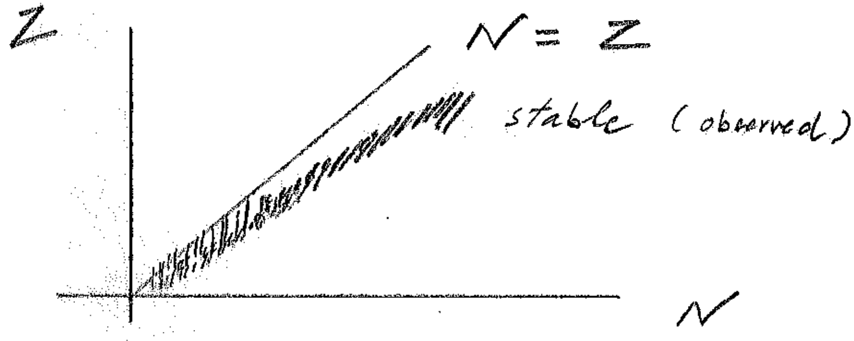


what binds n, p together?

Nuclear Chart

① isospin sym. $n \approx p$

② $N > Z$ for larger nucleus



$$\Delta E_C \sim \frac{3}{5}\frac{(Ze)^2}{4\pi R}$$

$$R = r_0 A^{1/3}$$

$$\Delta E_C \sim Z^2 A^{-1/3}\left(\frac{3}{5}\frac{\alpha}{r_0}\right)$$

Liquid Drop Model

a_C empirical: 0.71 MeV

$\leftrightarrow$ $r_0 = 1.2$ fm

the attractive nuclear force needs to beat that: ←

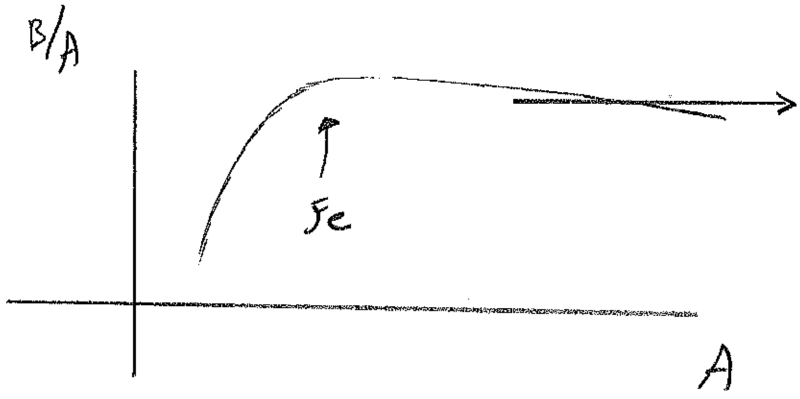
$-a_V A$

$a_V \sim 15.85$ MeV

w huge corr from surface effect

Fermi Gas

$n \approx p$ to $\downarrow E_F$



saturation

~ 8 MeV

hint at the short range nature of nuclear force

if the attractive nuclear force is ∞ long range

$\longrightarrow \quad \Delta E_V \propto -\frac{1}{2} A(A-1) \sim -A^2$

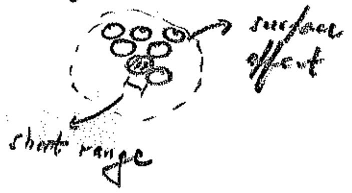
$B/A \sim \propto A$ instead of $B/A \propto 1$: observed // Nuclear saturation //

$\longrightarrow$ Yukawa's idea : short range force is mediated by a massive bosons $\longrightarrow$ pions.

$r_0 \to 1.2\ fm \iff E = 0.16\ GeV.$

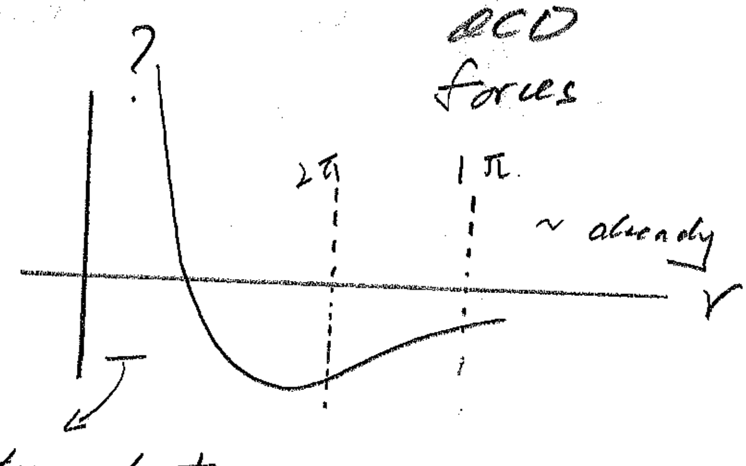
Nucleons pack together

$m_\pi \to 0.14\ GeV$



scale of residual QCD forces

eventually got discovered after μ ~ 106 MeV came to confuse us.



~ already "short"

ultra-short distances

Liquid Drop Model

$$M_{Z,A} \longrightarrow Z m_p + (A-Z) m_n$$

$$- a_V A + a_S A^{2/3} +$$

$$a_C Z^2 A^{-1/3} + a_A \frac{(Z - \frac{A}{2})^2}{A} +$$

$$- \frac{(-1)^Z + (-1)^{A-Z}}{2} a_P A^{-1/2}$$

$$B_{Z,A} \longrightarrow Z m_p + (A-Z) m_n - M_{Z,A}$$

$a_V = 15.85$ MeV — Nuclear saturation

$a_S = 18.34$ MeV — surface effects

$a_C = 0.71$ MeV — Coulomb.

$a_A = 92.86$ MeV — Asymmetry: prefer $Z = N = \frac{A}{2}$

$a_P = 11.46$ MeV — Pairing ~ isotopes

even $A \rightarrow$

N	Z
odd	odd
even	even

odd $A \rightarrow$ odd $\leftrightarrow$ even

$R \sim A^{1/3}$

$S \sim R^2 \sim A^{2/3}$

We can do many physics with it:

Isotopes:

Z - dependence at fixed A:

$$\left.\frac{\partial M}{\partial Z}\right|_{A \text{ fixed}} = 0 \Rightarrow Z_0(A) \text{ line.}$$

if we neglect pairing:

$$0 = m_p - m_n + 2a_c Z A^{-1/3} + 2a_A \frac{Z - \frac{A}{2}}{A}$$

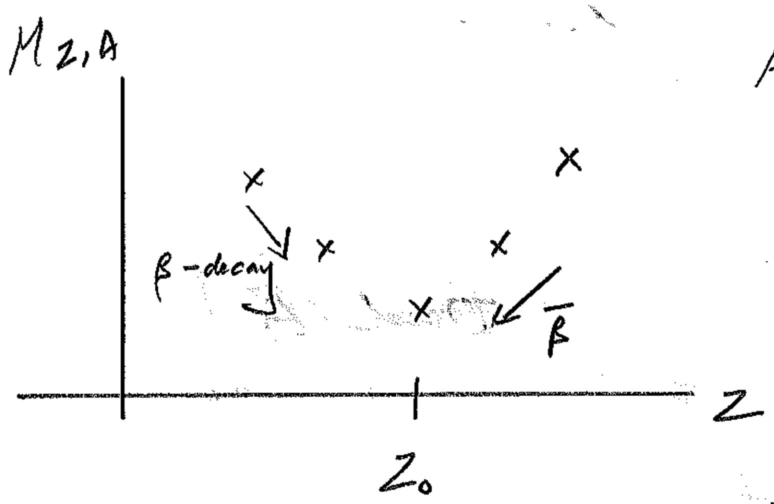
$$Z_0(A) = \frac{A}{2}\left[\frac{m_n - m_p + a_A}{a_c A^{2/3} + a_A}\right]$$

if $m_n = m_p$ & $a_c \to 0$

$Z_0(A) = A/2$ as expected

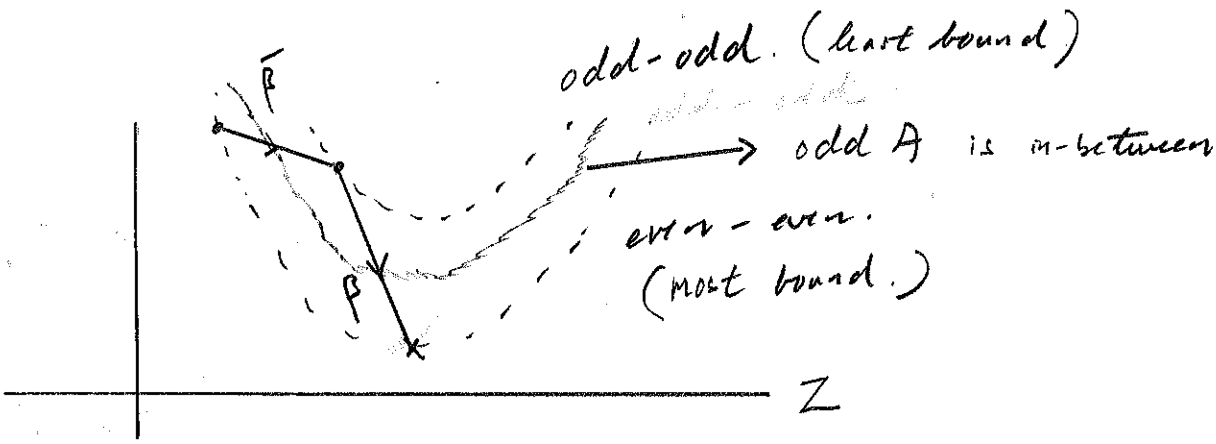
if $A \uparrow \Rightarrow Z_0 < \frac{A}{2}$ to avoid paying Coulomb

A = odd



A = even

w pairing



→ isospin effects

nn, pp loves to bind

→ $\underline{I} = 1$ is preferred

vs $\underline{I} = 0$

liquid drop model $\longleftrightarrow$ collective model

$\lambda^{NM}_{mfp} \gg$ size of nucleus   few $\to$ 10 fm.

$\leftrightarrow$ strongly interacting, quantum liquid

λ^{NM}_{mfp} ↗ ↙ (II) Nuclei

very quantum.

Vs classical liquid

$\lambda^{water}_{mfp} \ll$ size of water droplet

not really a good estimate $\longrightarrow$ $\lambda^{NM}_{mfp} \sim \dfrac{1}{n\,\sigma}$ $n \sim 0.16\ fm^{-3}$

$\sim 1\,fm$ $\sigma_{vac} \sim \alpha \dfrac{1}{m_\pi^2} \sim 6.22\,fm^2$

$\gtrsim R$ but $\sigma_{eff} \downarrow \longrightarrow 1-3\ fm^2$

phase space reduction due to Pauli's blocking

So quantum $\longrightarrow$ that it becomes quite simple

$\downarrow$

Free Fermi Gas.

Fermi Gas Model

Why free gas ?

if ∀ levels are filled ~ Pauli's blocking

by Pauli's exclusion

→ there is no place to go
if they would interact

$$E_{n_x n_y n_z} = \frac{1}{2m} \frac{\pi^2}{L^2} \left(n_x^2 + n_y^2 + n_z^2 \right)$$

↳ 1, 2, ...

$$dN_k = \frac{1}{8} \left(\frac{L}{\pi} \right)^3 4\pi k^2 \, dk \qquad \cdot \, d_g$$

↳ degen = 2 for spin ½

$$N(k_F) = \frac{L^3}{3\pi^2} k_F^3$$

↓

$$N_n = V \frac{k_F^3(n)}{3\pi^2}$$

separate Fermi surfaces.

$$N_p = V \frac{k_F^3(p)}{3\pi^2}$$

$$E_i^K \xrightarrow{\text{nm-ed.}} \nu \int \frac{d^3k}{(2\pi)^3}\, 2\, \frac{k^2}{2m_N}\, \theta(k_{F_i}-k)$$

$$= \underline{\frac{3}{5}\frac{k_{F_i}^2}{2m_N} N_i(k_F)}$$

$$\hookrightarrow E_F = \frac{k_F^2}{2m_N}$$

$$E_{F_i} = \frac{k_{F_i}^2}{2m_N}$$

$$= \frac{(3\pi^2 n_i)^{2/3}}{2m_N}$$

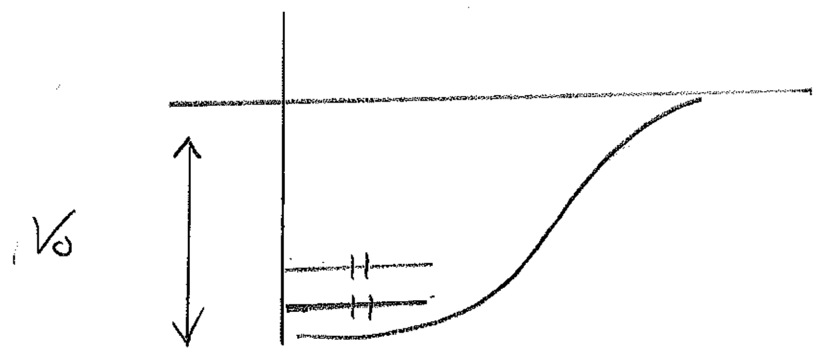
$Z = N = \frac{A}{2}$

if $n_i \to \bar{n}/2$

$\bar{n} = 0.16\ \text{fm}^{-3}$

$\frac{3}{5} E_{F_i} \sim 22$ MeV at saturation

we need to introduce $-V_0$ to bind them.



$$-a_V \mapsto -(V_0 - 22\,\text{MeV}).$$

$\wr$

$\boxed{16\ \text{MeV}}$

Binding pot strength:

$V_0 \sim 38$ MeV

why not 8 MeV? $\longrightarrow$ surface effect & coulomb ...

$$\Delta E_{Z,A} = E_n^K + E_p^K - V_0 A$$

$$\frac{3}{5}(A-Z)E_F(n) \qquad \frac{3}{5} Z E_F(p)$$

$$E_F(p) = \frac{k_F(p)^2}{2m_N} \rightarrow \left(\frac{3\pi^2 Z}{V}\right)^{2/3} \frac{1}{2m_N}$$

$$= \frac{1}{2m_N}(3\pi^2 \bar{n})^{2/3} \left(\frac{Z}{A}\right)^{2/3}$$

$$E_F(n) = \frac{1}{2m_N}(3\pi^2\bar{n})^{2/3}\left(\frac{A-Z}{A}\right)^{2/3}$$

$$\Delta E_{Z,A} = \frac{3}{5}\frac{1}{2m_N}(3\pi^2\bar{n})^{2/3}\left\{\underbrace{\frac{Z^{5/3}}{A^{2/3}} + \frac{(A-Z)^{5/3}}{A^{2/3}}}_{\frac{1}{2^{2/3}} A \left\{1 + \frac{20}{9}\frac{\Delta^2}{A^2} + \cdots\right\}}\right\} - V_0 A$$

where $\Delta = Z - \frac{A}{2}$

$$= \left[\underline{\frac{3}{5}\frac{1}{2m_N}\left(3\pi^2\frac{\bar{n}}{2}\right)^{2/3}} - V_0\right] A +$$

$$\underline{\frac{3}{5}\frac{1}{2m_N}\left(3\pi^2\frac{\bar{n}}{2}\right)^{2/3}}\;\frac{20}{9}\;\frac{\Delta^2}{A}$$

$-a_V$

$= (22\text{ MeV} - V_0)$

$a_A \sim 50$ MeV

only half of best fit, but ok...

Fermi Gas :

$$\Delta E_{Z,A} \rightarrow \Big(\underbrace{\tfrac{3}{5}\left(3\pi^2 \tfrac{\bar{n}}{2}\right)^{2/3} \tfrac{1}{2m_N} - V_0}_{\sim\, -a_V}\Big)\, A$$

$$+ \ \frac{3}{5}\left(3\pi^2 \frac{\bar{n}}{2}\right)^{2/3} \frac{1}{2m_N}\ \frac{20}{9}\ \frac{(Z-\frac{A}{2})^2}{A}$$

$$+ \ Z^2 A^{-1/3}\ \frac{3}{5}\frac{\alpha}{r_0} \ + \ \ldots$$

$$r_0 \rightarrow 1.2 \text{ fm}.$$

Quantum Free Gas gets us quite far

$\longrightarrow$ asymmetry E $\leftrightarrow$ n, p have separate Fermi surfaces

$\Rightarrow$ if $n = p$.

$n_n \rightarrow \frac{1}{2}\bar{n}$

$k_F(n/p) \rightarrow 260$ MeV

$E_F(n/p) \sim 37$ MeV

Symmetric matter

For pure neutron matter (NS).

$A = N \quad Z \rightarrow 0.$

$k_F(n) \rightarrow 331$ MeV

$E_F(n) \rightarrow 58$ MeV

at saturation $\longrightarrow$ will not bound if not for gravity

NS $\neq$ Giant Nucleus

01.2026

How to model Nuclear Matter ?
(T = 0) ①

Learn from the Liquid Drop Model

$$M_{Z,A} = m_N A + E_n^k + E_p^k - V_0 A + \dots$$

$\rightarrow$ KE + PE

include asym. $\rightarrow$ isospin dependence

pls do better than $-V_0 A$!

$$E_{NM} = V\int \frac{d^3p}{(2\pi)^3} 2\varepsilon_p \hat{n}_p + \underline{\mathcal{U}_{pot}[\hat{n}]}$$

↳ Skyrme potential model

En-functional

Ward-Luttinger

Φ-derivable

DFTs ...

thermodynamics :

$$dE = -\underline{p}dV + \mu dN + \cancel{TdS} \quad T=0$$

$$\mu = \left(\frac{\partial E}{\partial N}\right)_{V,S}$$

$$\underline{p} = \left(-\frac{\partial E}{\partial V}\right)_{N,S}$$

Now if we do fixed V

$\longrightarrow$ 1 variable : $n = \frac{N}{V}$

$$\mathcal{E} := \frac{E}{V} \longrightarrow d\mathcal{E} = \mu\, dn.$$

$$\mu(n) = \frac{\partial \varepsilon}{\partial n}$$

$$\mathcal{E} = -\underline{P} + \mu n = \int^n dn' \, \mu(n')$$

$\underline{P}$ is Legendre transform of ε

$$\mu = \frac{\partial \varepsilon}{\partial n} \longleftrightarrow n = \frac{\partial P}{\partial \mu}$$

$$\underline{P} = \left(n \frac{\partial}{\partial n} - I\right) \varepsilon \quad \text{in } n\text{-pic.}$$

$\mathcal{E}(n)$ is called the eq^{n}. of state
EOS

e.g. NR Fermi Gas :

$$\mathcal{E}(n) = \frac{3}{5} n \frac{p_F^2}{2m_N} \quad \text{w} \quad n = \frac{p_F^3}{3\pi^2}$$

$$\longrightarrow \frac{3}{5} \frac{1}{2m_N} (3\pi^2)^{2/3} \, n^{1+\frac{2}{3}}$$

$$\underline{P} = \left(\frac{5}{3} - 1\right) \varepsilon$$
$$= \frac{2}{3} \mathcal{E}$$

Polytropic index 5/3.
$\longrightarrow$ MR curve w Lane-Emden

$\mu \rightarrow \frac{5}{3} \frac{\varepsilon}{n} \longleftrightarrow$ it is just ε_F as verified from $E = \frac{3}{5} N \varepsilon_F$.

What about interacting Fermi Gas ?

$$\frac{E}{V} = \varepsilon = \int \frac{d^3p}{(2\pi)^3} \, 2\,\varepsilon_p \, \hat{n}_p \; +$$

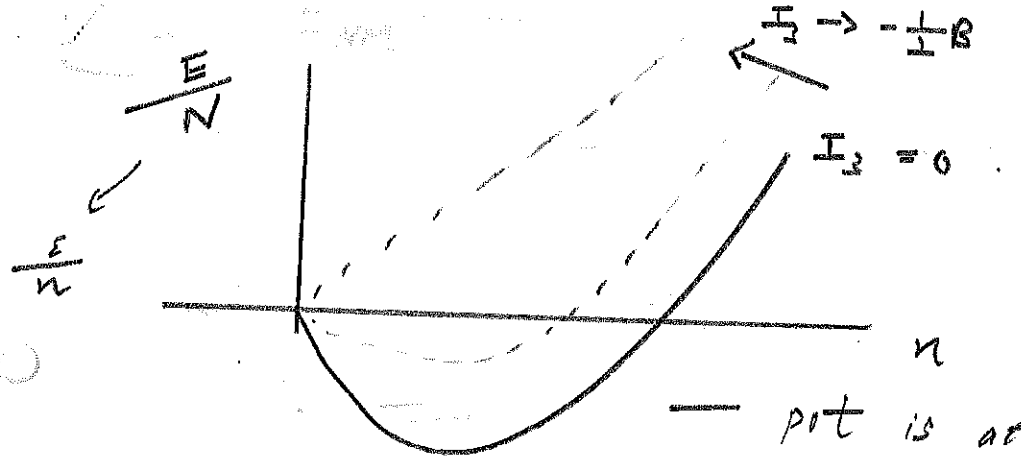
$\hat{n}_p \hookrightarrow \theta_{p_F - |\vec{p}|}$

$\mathcal{U}^{int}[n]$ (Long Story !)

model: $$\int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \, \hat{n}_{p_1} \hat{n}_{p_2} \, \underline{V_{\vec{p}_1 - \vec{p}_2}}.$$

→ needs some attractive part

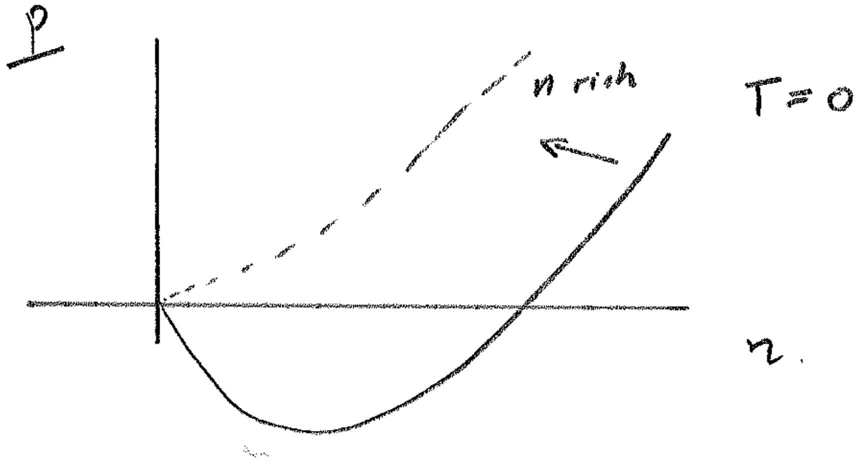
Focus on the pot.

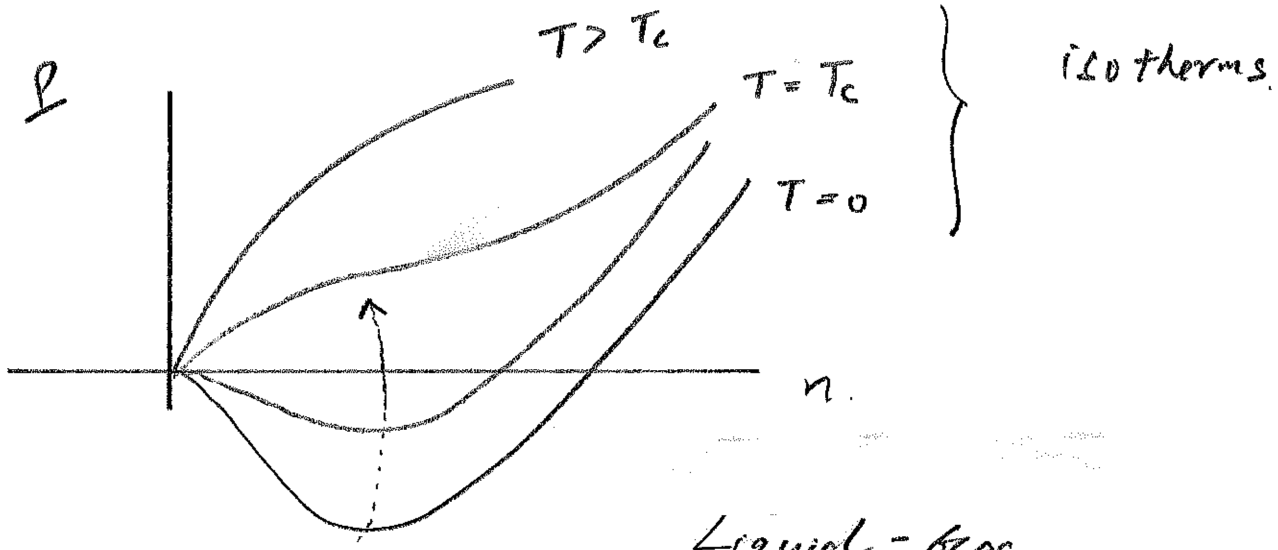


— pot is attractive

$$I_3 = \frac{1}{2}(N_p - N_n)$$

$$B = N_n + N_p.$$





Liquid - Gas transition → probed by HI/Cs.

11.2025

①

NN scattering via 1-pion exchange

isospin : $\vec{\tau} = \frac{1}{2}\vec{\sigma}$

$$\mathcal{L}_{int} = g_{\pi NN} \left(\bar{N}_x \, i\gamma_5 \, 2\vec{\tau}_I \cdot \vec{\pi}_{(x)} \, N_x \right)$$

Goldburger-Treiman relation : $\frac{g_A m_N}{f_\pi}$

isospin :

$N = \begin{pmatrix} p \\ n \end{pmatrix}$ as dictated by $\sigma_3 = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}$

$$\gamma^0 \to \begin{bmatrix} I & \\ & -I \end{bmatrix}$$

$$\gamma_5 \to \begin{bmatrix} & I \\ I & \end{bmatrix}$$

Dirac :

$$u_{\vec{p}s} = \sqrt{\frac{E+m_N}{2m_N}} \begin{bmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m_N}\chi_s \end{bmatrix} \xrightarrow{\text{non-rel. limit}} \begin{bmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{2m_N}\chi_s \end{bmatrix}$$

Dirac structure

Isospin structure

$$\pi^{\pm} := \frac{1}{\sqrt{2}}(\pi_1 \mp i\pi_2)$$
$$\pi^0 := \pi_3$$

$$2\vec{\tau}_I \cdot \vec{\pi} = \begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{bmatrix}$$

$$\mathcal{L}_{int} \to g_{\pi NN} \Big(\sqrt{2}\, \bar{p}\, i\gamma_5 n\, \pi^+ + \sqrt{2}\, \bar{n}\, i\gamma_5 p\, \pi^- + \bar{p}\, i\gamma_5 p\, \pi^0 - \bar{n}\, i\gamma_5 n\, \pi^0 \Big)$$

π^+ : kill π^+ / create π^-

by construction we get

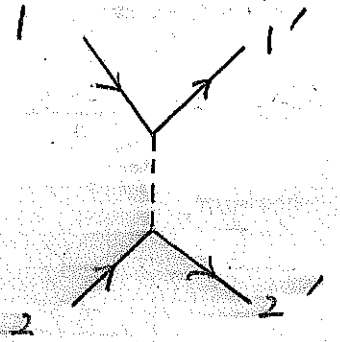
$$g_{\pi^+ n ; p} = g_{\pi^- p ; n} = \sqrt{2}\, \underline{g_{\pi^0 p ; p}} = \sqrt{2}\, g_{\pi^0 n ; n}$$

↑ larger

↳ our choice of $g_{\pi NN}$.

which is verified experimentally

Dirac structure :



$iM \leftrightarrow$

$$\langle 1'2' | \frac{1}{2} i^2 \int \mathcal{L}_{int} \int \mathcal{L}_{int} | 1 2 \rangle$$

$$\langle 1'2' | \frac{1}{2} i^2 g_{\pi NN}^2 \int \bar{N}_x i\gamma_5 2\tau^a N_x \pi_x^a \int \bar{N}_y i\gamma_5 2\tau^b N_y \pi_y^b | 1 2 \rangle$$

$$= -\frac{1}{2} g_{\pi NN}^2 \; \bar{u}_{1'}\gamma_5 u_1 \; \bar{u}_{2'}\gamma_5 u_2 \; \pi_x^a \pi_y^b$$

$$\left(-4\, \tau^a_{1'1}\, \tau^b_{2'2}\right) \times \quad \text{sign}\,(1'1\; 2'2)$$

$\times$ 2
sym. fac
$x \leftrightarrow y$

↙ + permutation from 2'1'1 2

$$\langle \pi_x^a \pi_y^b \rangle \longleftrightarrow \delta^{ab}\, i D_\pi$$

$$\hookrightarrow \frac{i}{q^2 - m_\pi^2 + i\delta}$$

$$\longrightarrow -i V_\pi$$

$$V_\pi = \frac{1}{\vec{q}^{\,2} + m_\pi^2}$$

$$q = p_1 - p_1' = p_2' - p_2$$

$$\bar{u}_{1'} \gamma_5 u_1$$

$$= \left(\chi_{1'}^\dagger \quad \chi_{1'}^\dagger \frac{\vec{\sigma}\cdot\vec{p}_1{}'}{2m_N} \right) \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \begin{pmatrix} \chi_1 \\ \frac{\vec{\sigma}\cdot\vec{p}_1}{2m_N}\chi_1 \end{pmatrix}$$

$$= \chi_{1'}^\dagger \vec{\sigma} \chi_1 \cdot \frac{(\vec{p}_1 - \vec{p}_1{}')}{2m_N}$$

$$= \chi_{1'}^\dagger \vec{\sigma} \chi_1 \cdot \frac{\vec{q}}{2m_N}$$

$$\bar{u}_{2'} \gamma_5 u_2 = \chi_{2'}^\dagger \vec{\sigma} \chi_2 \cdot \frac{\vec{p}_2 - \vec{p}_2{}'}{2m_N} = -\chi_{2'}^\dagger \vec{\sigma} \chi_2 \cdot \frac{\vec{q}}{2m_N}$$

$$iM = i \frac{g_{\pi NN}^2}{4M_N^2} \, 4\, \tau^a_{1'1} \tau^a_{2'2} \, \frac{\vec{\sigma}_{1'1}\cdot\vec{q} \;\; \vec{\sigma}_{2'2}\cdot\vec{q}}{\vec{q}^{\,2} + m_\pi^2}$$

$\updownarrow$ Born amplitude

$$-iV_{pot}$$

$$g_{\pi NN} \longleftrightarrow \left(\frac{g_A}{2f_\pi}\right)^2$$

$$V_{pot} = -\frac{g_{\pi NN}^2}{4m_N^2} \, \sigma_1^I \cdot \sigma_2^I \, \frac{\sigma_1\cdot\vec{q} \;\; \sigma_2\cdot\vec{q}}{\vec{q}^{\,2} + m_\pi^2}$$

pseudoscalar structure $\rightarrow \bar{u}\gamma_5 u \rightarrow \vec{\sigma}\cdot\vec{q}$ term

tensor force!

tensor structure:

$$\sigma_1\cdot\vec{q}\;\sigma_2\cdot\vec{q}$$

$$= \sigma_1^i\sigma_2^j\, q^i q^j$$

$\frac{1}{3}S_{12}^{ij}$

traceless, rank 2 tensor

$$= \left[\frac{1}{3}\delta^{ij}\,\sigma_1\cdot\sigma_2 + \left(\sigma_1^i\sigma_2^j - \frac{1}{3}\delta^{ij}\,\sigma_1\cdot\sigma_2\right)\right]$$

$$\left[\frac{1}{3}\delta^{ij}\,\vec{q}^{\,2} + \left(q^i q^j - \frac{1}{3}\delta^{ij}\,\vec{q}^{\,2}\right)\right]$$

$\frac{1}{3}\alpha^{(2)\,ij}$

$$= \frac{1}{3}\,\sigma_1\cdot\sigma_2\,\vec{q}^{\,2} +$$

$$\frac{1}{3}S_{12}^{ij}\left(q^i q^j - \underline{\frac{1}{3}\delta^{ij}\vec{q}^{\,2}}\right)$$

vanishes.

$$\longrightarrow \frac{1}{3}\,\sigma_1\cdot\sigma_2\,\vec{q}^{\,2} + \frac{1}{3}S_{12}^{ij}\,q^i q^j$$

$$\frac{\sigma_1\cdot\vec{q}\;\;\sigma_2\cdot\vec{q}}{\vec{q}^2+m_\pi^2}$$

$$\to \frac{1}{3}\,\sigma_1\cdot\sigma_2\,\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} \;+$$

$$\frac{1}{3}\,S_{12}^{ij}\;\;\frac{q^i q^j}{\vec{q}^2+m_\pi^2}$$

$\downarrow$ Fourier transform to $\vec{r} = \vec{x}_1-\vec{x}_2$ space

$$\frac{1}{3}\,\sigma_1\cdot\sigma_2\;\;m_\pi^2\;\left(-V_\pi(r) + m_\pi^{-2}\,\delta^3_{\vec{r}}\right)$$

$$+$$

$$\frac{1}{3}\,S_{12}^{ij}\;\;-\partial_i\partial_j\;\;V_\pi(r)$$

where $V_\pi(r) = \dfrac{e^{-m_\pi r}}{4\pi r}$

for <u>any</u> radial function $f(r)$

$$\partial_i\partial_j f(r) = \partial_i\left(f'\frac{x^j}{r}\right)$$

$$= \left(f''-\frac{1}{r}f'\right)\frac{x^i x^j}{r^2} + \underline{\frac{f'}{r}\,\delta^{ij}}$$

will $\to 0$ when dotted with S_{12}^{ij}

for us:

$$V_\pi''-\frac{V_\pi'}{r} \to m_\pi^2\,V_\pi(r)\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)$$

$$S_{12}^{ij}\;\frac{x^i x^j}{r^2} \to S_{12}(\hat{r}) = 3\,\sigma_1\cdot\hat{r}\;\sigma_2\cdot\hat{r} - \sigma_1\cdot\sigma_2$$

$$V_{pot} = -\left[\frac{g^2_{qNN}}{4 m_N^2} m_q^2\right] \overset{\nearrow \sim 1}{} \frac{1}{3} \times$$

$$\sigma_1^I \cdot \sigma_2^I \left\{ \sigma_1 \cdot \sigma_2 \left(-V_q(r) + m_q^{-2} \delta^3(\vec{r})\right) \right.$$

$$\left. + \ S_{12}(\hat{r}) \left(-V_q(r)\right)\left(1 + \frac{3}{m_q r} + \frac{3}{m_q^2 r^2}\right) \right\}$$

Isospin structure

$$\sigma_1^I \cdot \sigma_2^I = 4 \left(\tau_1^I \cdot \tau_2^I\right)$$

$$= 2\left[\left(\tau_1^I + \tau_2^I\right)^2 - \tau_1^2 - \tau_2^2\right]$$

$$= 2\left[\, I(I+1) - \frac{1}{2}\frac{3}{2} \times 2 \,\right]$$

$$= \begin{cases} -3 & I = 0 \quad \text{singlet} \\ 1 & I = 1 \quad \text{triplet} \end{cases}$$

similarly for spins

$$\sigma_1 \cdot \sigma_2 \rightarrow \begin{cases} -3 & S = 0 \\ 1 & S = 1 \end{cases}$$

N's are spin ½

$$S_{12} \to 3\,\sigma_1\cdot\hat{r}\;\sigma_2\cdot\hat{r} - \sigma_1\cdot\sigma_2$$

$$\sim\; S^{(2)}\cdot R^{(2)}$$

$$\updownarrow$$

$$\left(x^i x^j - \frac{1}{3} r^2 \delta^{ij}\right)\frac{1}{r^2}$$

$\Rightarrow$ mixing L : $\Delta l = 2$ will mix

S & D waves

P & F etc

exp.

$$\frac{g^2_{\pi NN}}{4\pi} = 13.8 \quad \longleftrightarrow \quad g_{\pi NN} \sim 40\text{–}50 \text{ of } e$$

$\updownarrow$

$g_A \sim 1.31 \longleftrightarrow$ Nucleon is NOT elementary

$$\updownarrow$$

$$g_{\pi NN} = \frac{g_A M_N}{f_\pi}$$

$$\left[\frac{g^2_{\pi NN}}{4M_N^2}\frac{1}{3}M_\pi^2\right] \longleftrightarrow \frac{g_A^2 M_\pi^2}{12 f_\pi^2} \simeq \frac{1}{3}$$

the combination

$\updownarrow$

pion physics is shouting!

PWA :

NN $\rightarrow$ overall Antisymmetric by Pauli's exclusion

L	S	I	Anti-sym?
even	0	1	✓
even	1	0	✓
odd	0	0	✓
odd	1	1	✓

$^{2S+1}L_J$ $\quad$ $^1S_0$ $\quad$ $^3S_1$

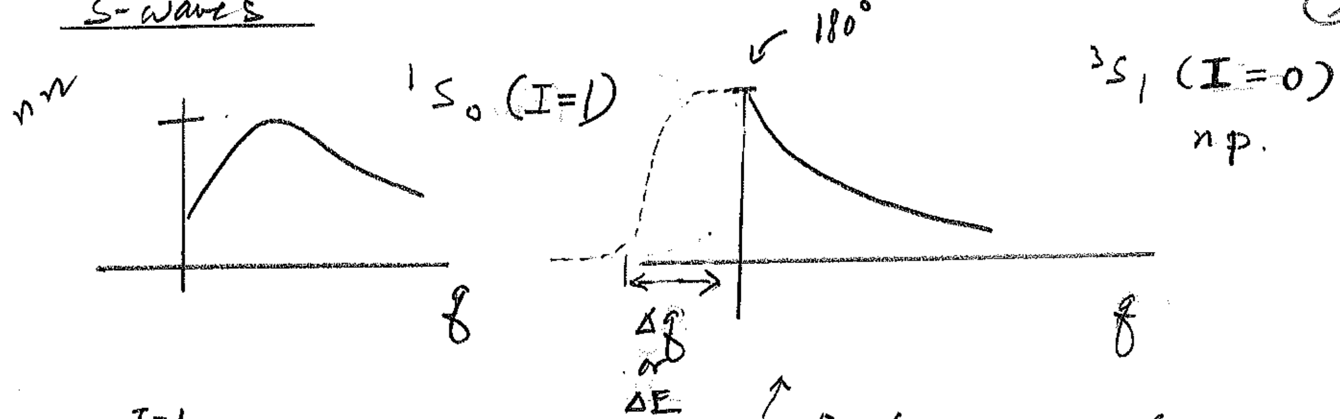
$J = (L+S)$ is conserved not L/S separately $\quad$ $^1P_1$ $\quad$ $^3P_0$ $\quad$ $^3P_1$ $\quad$ $^3P_2$

$^1D_2$ $\quad$ $^3D_1$ $\quad$ $^3D_2$ $\quad$ $^3D_3$.

$\leftrightarrow$ allowed states in $I = 1$

$\rightarrow$ nn, pp scat

S-waves



↑ Deuteron channel

$a_{nn}^{I=1} \sim +18$ fm

$a_{np}^{I=1} \sim +23.7$ fm.

$\delta \approx qa + \cdots$

following ...

" my convention for a "

$a < 0$ means repulsion.

$a > 0$ means attraction

$I = 0$ (n-p BS.)

parity $(-1)^L \sim +$

$^3S_1 \xleftrightarrow{\text{mixed w}} {}^3D_1$

$\Delta l = 2$

fixed I can mix.

$a_{np}^{I=0} \sim -5.4$ fm $\sim$ residual. repulsive

D: weakly bound : $\Delta E \sim 2.2$ MeV. shallowly bonded

$|a_{I=1}| \gg 1$ fm

it is INSANELY large !

$\longrightarrow$ luckily : lots of cancellation

$a \sim 20$ fm $\longrightarrow$ longest length scale

$\longrightarrow$ min. E scale $\sim$ 10 MeV

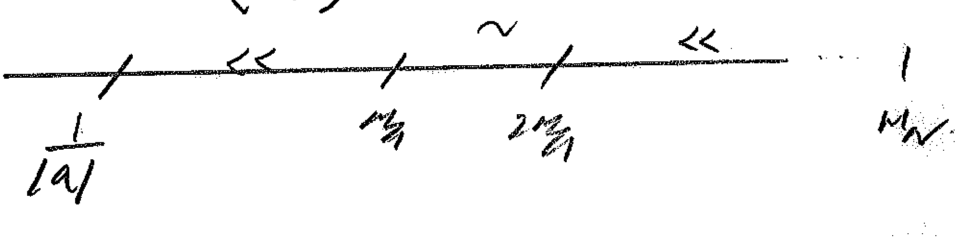
$m_\pi \sim 140$ MeV

$M_\Delta - M_N \sim 2 m_\pi$

$M_N \sim 1$ GeV.

don't see the pions → ERE scat etc. ↔ pions are "heavy" → contact ints ... fun.

separation of scales ...



P-waves ?

⟶ nice physics. of spin-orbit. coupling

$^3P_0$ $\quad$ $^3P_1$ $\quad$ $^3P_2$.

amazing splitting.

$\langle \vec{L}\cdot\vec{S} \rangle$: -2 $\quad$ -1 $\quad$ 1

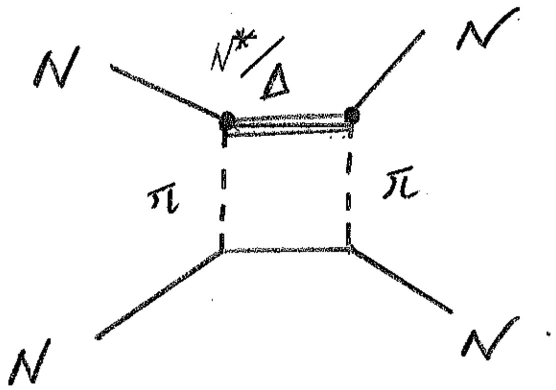
$$\frac{1}{2}\left(J^2 - L^2 - S^2\right)$$

$L^2 \to 1(2)$, $\quad S^2 \to 2$

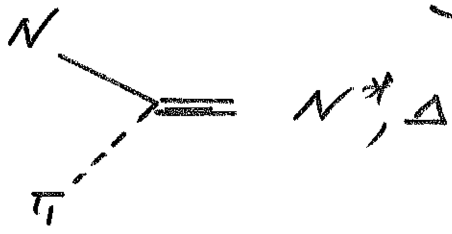
Beyond 1-pion exchange

↓ for shorter distance / high E

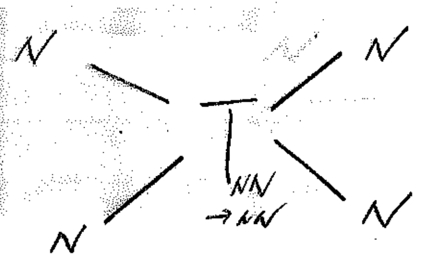
exchange 2 pions

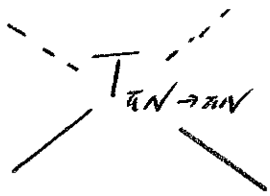


need to study

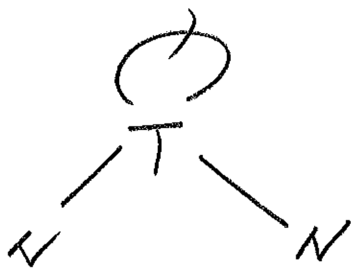


πN phase shifts





Σ_N =



$+ \cdots$ (optical potential)

$$\frac{e^{i\delta}}{q}\sin\delta$$

$$\Sigma_N \sim 2M_N V_{opt} \leftrightarrow 2M_N \frac{-4\pi}{2M_{red}} n_X f_{NX\to NX}$$

$$\hat{=} \; -4\pi n_X f$$

if scatter X is extremely heavy → static

→ Scattering theory

Walecka Model ? → not qualified as a "meson" (pt-like) to be exchanged.

$2\pi \sim \sigma \rightarrow n_s^2$
$3\pi \sim \omega \rightarrow n_v^2$
} bad physics.

$\omega \rightarrow \rho\pi$ states

$\rho \rightarrow \pi\pi$ states

too short range to use meson exchange pic.

∞ hard to define exchange when they stick together ...

Surprising → dominant model for nucl. matter ...

Density functional approach

$$\mathcal{L} \rightarrow \bar{\psi}(i\partial\!\!\!/ - m_N)\psi - \mathcal{U}[n_v, n_s]$$

confinement ?

NS
PNS

→ HICs

NM Vs Quark Matter

Why RMF is bad ?

→ okay if treated as a parametrization of isospin dependent nuclear force

→ but exchange of physical particles ?

Not really !

① 2π's $\leftrightarrow$ σ is not working

↳ broad, not well-defined resonances.

② ρ, ω $\leftrightarrow$ reasonable resonances

but

0.8 GeV $\leftrightarrow$ 0.25 fm

exch ?

Nucleons are touching

→ exch. model ~~fails~~

→ residual color forces

/ quark (gluon) sub-structure of Nucleons.

if RMF param.

→ fit to nmat

they fail the vac.

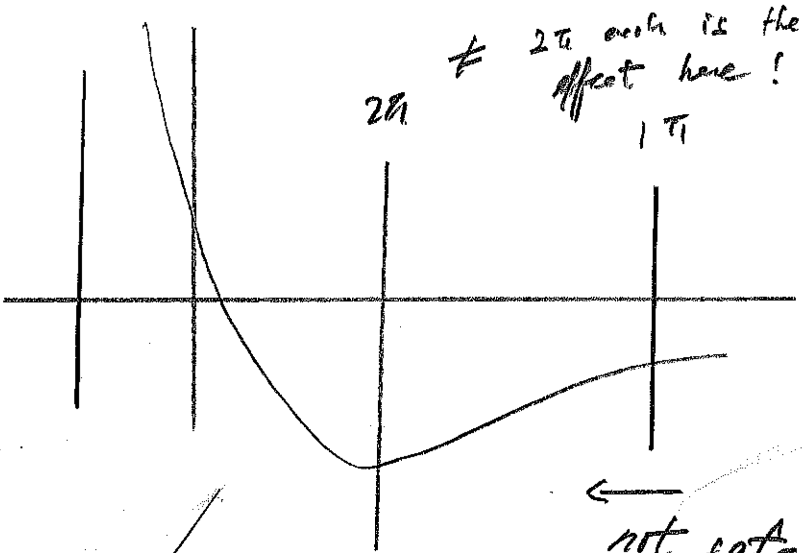
→ if they fit the vac

→ X explaining nmat ↑.

⟹ not reliable : need medium-dep. couplings ...

be careful in interpreting the result

≠ 2π exch is the only responsible effect here!



← not safe to consider 1PE.

V_{NN} → how this manifests in a many-body system?

↳ HF, B or DFT

03. 2025

Tensor construction

$A_i B_j$ 9 entries

$$A_i B_j = \frac{1}{2}(A_i B_j + A_j B_i) + \frac{1}{2}[A_i B_j - A_j B_i]$$

$$= \left(\frac{1}{3}\vec{A}\cdot\vec{B}\,\delta_{ij} + \mathcal{Q}_{ij}\right) + \frac{1}{2}\varepsilon_{ijk}(\vec{A}\times\vec{B})_k$$

Sym.

Anti-Sym.

where

$$\mathcal{Q}_{ij} = \frac{1}{2}\left[A_i B_j + A_j B_i - \frac{2}{3}(\vec{A}\cdot\vec{B})\,\delta_{ij}\right]$$

we identify 3 structures of the tensor

$R_q^{(0)} \sim \delta_{ij}\, r^2$ $\qquad 9 = 1 + 3 + 5$

$R_q^{(1)} \sim \varepsilon_{ijk}\, r^k$

$R_q^{(2)} \sim \frac{1}{2}(r_i r_j + r_j r_i) - \frac{1}{3}\vec{r}^{\,2}\delta_{ij}$

when we combine 2 tensors :

$$R_q^{(0)}\,\sigma_q^{(0)} \sim r^2\,\vec{\sigma}\cdot\vec{\sigma}$$

$$R_q^{(1)}\,\sigma_q^{(1)} \sim \vec{r}\cdot\vec{\sigma}$$

$$R_q^{(2)}\,\sigma_q^{(2)} \to \sum_q R_q^{(2)}\,\sigma_q^{(2)}\,(-1)^q\,.$$

$$\sim \left(\frac{r_i r_j + r_j r_i}{2} - \frac{1}{3} r^2 \delta_{ij}\right)\left(\frac{\sigma_i\sigma_j + \sigma_j\sigma_i}{2} - \frac{1}{3}\sigma^2\delta_{ij}\right)$$

$$= \vec{r}\cdot\vec{\sigma}\;\vec{r}\cdot\vec{\sigma} - r^2\vec{\sigma}^2\left(\frac{2}{3} - \frac{1}{9}3\right)$$

$$= \vec{r}\cdot\vec{\sigma}\;\vec{r}\cdot\vec{\sigma} - \frac{1}{3} r^2 \sigma^2$$

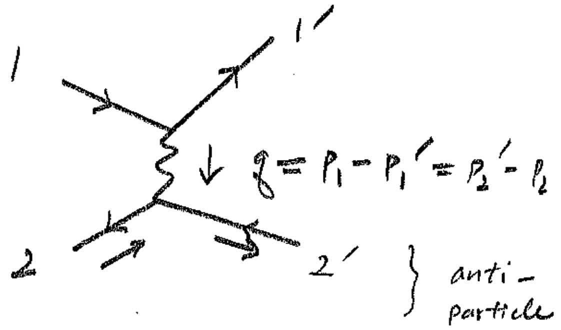
//

11. 2025

LS coupling :

$\vec{S}\cdot\vec{L} \rightarrow$ relativistic correction of scalar channel

$V_S \quad \bar{u}_{1'} u_1 \quad \bar{v}_2 v_{2'}$



$$\bar{u}_{1'} u_1 \rightarrow \chi_{1'}^{\dagger}\left[1 - \frac{\sigma\cdot\vec{p}_1{}'\ \sigma\cdot\vec{p}_1}{(E_1'+m_1)(E_1+m_1)}\right]\chi_1$$

$$\approx \chi_{1'}^{\dagger}\chi_1\left(1 - \frac{\vec{p}_1{}'\cdot\vec{p}_1}{4m_1^2}\right) +$$

$$- \frac{1}{4m^2}\, i\,\varepsilon^{ijk}\, p_1'{}^{i}\, p_1^{j}\, \sigma_1^{k}$$

using

$\sigma^i\sigma^j = \delta^{ij}\mathbb{1}_2 + i\varepsilon^{ijk}\sigma^k$

$\hookrightarrow (\underline{p_1} - q)^i$

not useful

$$\bar{v}_2 v_{2'} = \left(\chi_2^{c\dagger}\frac{\sigma\cdot\vec{p}_2}{E_2+m_2} \quad \chi_2^{c\dagger}\right)\begin{pmatrix}\frac{\sigma\cdot\vec{p}_2{}'}{E_2'+m_2}\chi_{2'}^{c} \\ -\chi_{2'}^{c}\end{pmatrix}$$

$$\approx -\left\{\chi_2^{c\dagger}\chi_{2'}^{c}\left(1 - \frac{\vec{p}_2\cdot\vec{p}_2{}'}{4m_2^2}\right)\right.$$

$$\left. - \frac{1}{4m_2^2}\, i\,\varepsilon^{ijk}\, p_2^{i}\, p_{2'}^{j}\,(-\sigma_2^{k})\right\}.$$

$\hookrightarrow (\vec{p}_2 + \vec{q})^j$

$\bar{u}_1' u_1 \quad \bar{v}_2' v_2$ CM frame

$$\longrightarrow \quad \underline{-\delta\delta} + \frac{-1}{4m_1^2} i\,\varepsilon^{ijk} q^i p_1^j \sigma_1^k$$

leading

$$+ \frac{1}{4m_2^2} i\,\varepsilon^{ijk} q^i p_2^j \sigma_2^k.$$

$p_2 \to -p_1$

$$\Delta V_s = -\frac{1}{4} i\,\varepsilon^{ijk} q^i p_1^j \left[\frac{\sigma_1^k}{m_1^2} + \frac{\sigma_2^k}{m_2^2}\right] V_s(q)$$

FT

$$-\frac{1}{4}\left(\frac{\vec{\sigma}_1}{m_1^2} + \frac{\vec{\sigma}_2}{m_2^2}\right)\cdot \vec{L} \; \frac{1}{r}\frac{d}{dr} V_s \qquad q^i \to -i\partial_i$$

$\vec{L} \leftrightarrow \vec{x}\times\vec{p}$

derivative interaction if $V_s(\underline{\underline{r}})$

$$q^i V_s \to -i V_s' \frac{x^i}{r}$$

$$\longrightarrow \quad \vec{S}_i = \frac{1}{2}\vec{\sigma}_i$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2 \longrightarrow S = 0 \quad \text{singlet}$$

$$S = 1 \quad \text{triplet}$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$\vec{L}\cdot\vec{S} \longrightarrow \frac{1}{2}\left[\underset{j(j+1)}{J^2} - \underset{l(l+1)}{L^2} - \underset{s(s+1)}{S^2}\right]$$

is the best recipe to handle this tensor