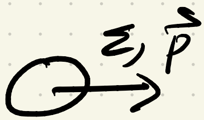
Relativistic Kinematics



$$E = \sqrt{\vec{p}^{\,2} + m^2}$$

$E^2 - \vec{p}^{\,2} = m^2$ is Lorentz Invariant
L.I.

$$E' = \gamma(E - \beta p_\parallel)$$

$$p_\parallel' = \gamma(p_\parallel - \beta E)$$

$$\vec{p}_T{}' = \vec{p}_T$$

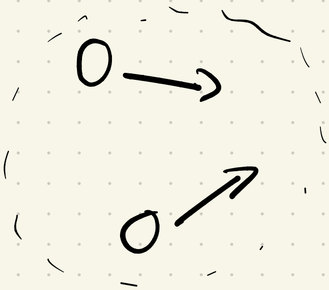
$$E^2 - p_\parallel^2 = \vec{p}_T^{\,2} + m^2 = m_T^2$$

is "L.I." (not really)

$$E = m_T \cosh y$$

$$p_\parallel = m_T \sinh y$$

2-particle system



$$\underline{P} = E, \underline{\vec{p}}$$

$$E = E_1 + E_2$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

$$E^2 - \vec{P}^2 = s \quad \text{is L.I.}$$

$\vec{P}^* = \vec{0}$: CM frame

$$E^* = \sqrt{s} \qquad \vec{P}^* = 0 = \vec{p}_1^* + \vec{p}_2^*$$

$$\vec{q} = \vec{p}_1^*$$

$$\sqrt{s} = \sqrt{\vec{q}^2 + m_1^2} + \sqrt{\vec{q}^2 + m_2^2}$$

$$\Rightarrow \quad q = \frac{1}{2}\sqrt{s}\sqrt{1 - \frac{(m_1+m_2)^2}{s}}\sqrt{1 - \frac{(m_1-m_2)^2}{s}}$$

//

also

$$E_1^* = \frac{s - m_2^2 + m_1^2}{2\sqrt{s}}$$

depends on s!

$$E_2^* = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}$$

↙

how to derive that?

$p_1 \cdot P$ is L.I.

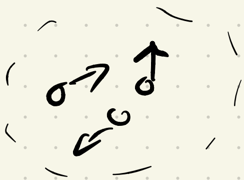
$= E_1^* \sqrt{s}$ in CM frame

$$E_1^* \sqrt{s} = p_1 \cdot P = -\frac{1}{2}\Big[\overbrace{(p_1 - P)^2}^{m_2^2} - \underbrace{p_1^2}_{m_1^2} - \underbrace{P^2}_{s}\Big]$$

$$\Rightarrow \quad E_1^* = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}$$

$$E_2^* = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}} \quad //$$

N-body



$$\underline{P} = \sum_i p_i$$

$$\underline{P} = [E, \vec{P}]$$

$$E = \sqrt{\vec{P}^2 + \sqrt{s}^2}$$

still can define s for the system!

$$\Phi_N(s) := \int \prod_i \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \times (2\pi)^4 \delta^4\left(\underline{P} - \sum_i p_i\right)$$