10.2023

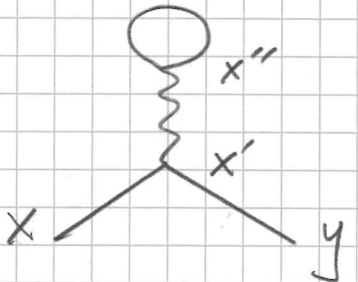
Translational Invariance
&
Feynman's Rules

Convention :

$$\varphi(x) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot x}\, \tilde{\varphi}(k)$$

$$\tilde{\varphi}(k) = \int d^4x\, e^{ik\cdot x}\, \varphi(x)$$

Hartree diagram



(lazy G_{xy} for $x \to y$ instead of $f \leftarrow i$ convention ...)

$$i\Delta G_{xy} = \int d^4x'\, d^4x''\; iG_{xx'}\, iG_{x'y} \times \left\{ (-ig)\, iU_{x'x''} (-ig)\, iG_{x''x''} (-1) \right\}$$

$$= g^2 \int d^4x'\, d^4x''\; G_{xx'}\, G_{x'y}\, U_{x'x''}\, \underbrace{G_{x''x''}}_{G_{x=0}}$$

$\downarrow$ p-space

$$i\Delta\tilde{G}(p) = g^2 G_{x=0} \int d^4x\, e^{ip\cdot(x-y)}\, d^4x'\, d^4x''\, \frac{d^4l_1}{(2\pi)^4} \frac{d^4l_2}{(2\pi)^4} \frac{d^4l_3}{(2\pi)^4}\; e^{-il_1\cdot(x-x')}\, e^{-il_2\cdot(x'-y)}\, e^{-il_3\cdot(x'-x'')} \times$$

$$\tilde{G}_{l_1}\, \tilde{G}_{l_2}\, \tilde{U}_{l_3}$$

recall

$$\int d^4x \; e^{i p\cdot x} = (2\pi)^4 \delta^4(p)$$

$\int d^4x''$ requires $l_3 = 0$

$\int d^4x'$ requires $l_1 - l_2 - l_3 = 0$

or

$l_2 = l_1$

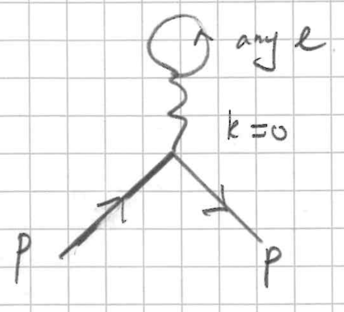
finally

$\int d^4x$ requires $l_1 \to p$.

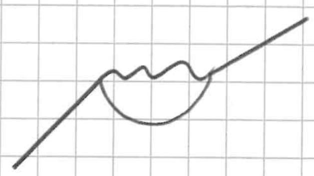
$$i\Delta \tilde{G}(p) = \underbrace{g^2 G_{x=0}}_{\int \frac{d^4l}{(2\pi)^4}\tilde{G}_l} \; \tilde{G}_p \; \tilde{U}_{k=0} \; \tilde{G}_p$$

$$\longrightarrow \; i\tilde{G}_p \left\{ (-ig)\, i\tilde{U}_{k=0} (-ig) \int_l i\tilde{G}_l \,(-1) \right\} i\tilde{G}_p$$

Which is just what you get when applying the Feynman's rule



Fock Diagram



$$i\Delta G_{xy} = \int d^4x'\, d^4x'' \; iG_{xx'}\,(-ig)\, iG_{x'x''}\,(-ig)\, iU_{x'x''} \,(iG_{x''y})$$

$$i\Delta\tilde{G}_p \equiv \int d^4x \; e^{ip\cdot(x-y)}\, d^4x'\, d^4x'' \frac{d^4l_1}{(2\pi)^4}\frac{d^4l_2}{(2\pi)^4}\frac{d^4l_3}{(2\pi)^4}\frac{d^4l_4}{(2\pi)^4}$$

$$(-1)\, g^2\; \tilde{G}_{l_1}\, \tilde{G}_{l_2}\, \tilde{U}_{l_3}\, \tilde{G}_{l_4} \;\times$$

$$e^{-il_1\cdot(x-x')}\, e^{-il_2\cdot(x'-x'')}\, e^{-il_3\cdot(x'-x'')}\, e^{-il_4\cdot(x''-y)}$$

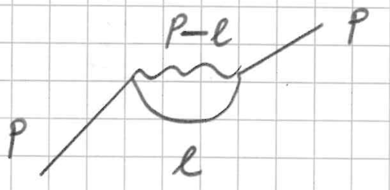
WOW, that's a lot! But easy!

$$\int d^4x' \longrightarrow l_1 = l_2 + l_3$$

$$\int d^4x'' \longrightarrow l_4 = l_2 + l_3 \longrightarrow = l_1$$

$$\int d^4x \longrightarrow l_1 = p$$

$l_1 = p$
$l_2 = l_2$
$l_3 = p - l_2$
$l_4 = p$



$$i\Delta\tilde{G}_p = -g^2\; \tilde{G}_p \left(\int \frac{d^4l_2}{(2\pi)^4}\; \tilde{G}_{l_2}\; \tilde{U}_{p-l_2}\right) \tilde{G}_p$$

$$= i\tilde{G}_p \left(\int_l (-ig)^2\, i\tilde{G}_l\, i\tilde{U}_{p-l}\right) i\tilde{G}_p \quad //$$

$$i\tilde{G}_p^{\text{Hartree}} = i\tilde{G}_p \left\{ (-ig)^2 \, i\tilde{U}_{k=0} \int_l i\tilde{G}_l (-1) \right\} i\tilde{G}_p$$

$$i\tilde{G}_p^{\text{Fock}} = i\tilde{G}_p \left\{ (-ig)^2 \int_l i\tilde{G}_l \, i\tilde{U}_{p-l} \right\} i\tilde{G}_p$$

$\Rightarrow$ relative $-$ sign between them

fermion loop