01. 2024

Deriving G.O.R. ulation

①

$$q \to e^{-i\gamma_5 \tau^a \alpha^a_{AV}} q$$

$$\Rightarrow \bar{q} \to \bar{q} e^{-i\gamma_5 \tau^a \alpha^a_{AV}}$$

$$j^{\mu a}_{AV} = \bar{q} \gamma^\mu \gamma_5 \tau^a q .$$

$$Q^a_{AV} = \int d^3x \, j^{0a}_{AV}$$

$$= \int d^3x \; \bar{q} \gamma^0 \gamma_5 \tau^a q$$

$$[Q^a_{AV}, H]$$

$$= [Q^a_{AV}, \int \bar{q} \hat{m} q]$$

<u>method 1</u>

$$Q^a_{AV} \int \bar{q} \hat{m} q - \int \bar{q} \hat{m} q \, Q^a_{AV}$$

$$= \iint (\bar{q} \gamma^0 \gamma_5 \tau^a q \; \bar{q} \hat{m} q - \bar{q} \hat{m} q \; \bar{q} \gamma^0 \gamma_5 \tau^a q)$$

Non-zero terms come from contractions

And there are 2

$$\int\int \bar{q}\,\gamma^0\gamma_5\tau^a\, q\, \bar{q}\,\hat{m}\, q$$

(1) (2)

$$\{q_x, q^\dagger_y\} = \delta_{xy}$$ → to be removed by integral

$$\{q, q\} = 0 = \{q, q^\dagger\}$$ otherwise.

(1) $\rightarrow \int \bar{q}\,\gamma^0\gamma_5\,\tau^a\,\gamma^0\,\hat{m}\,q.$

(2) $\rightarrow \int \gamma^0\gamma^0\gamma_5\,\tau^a\, q\,\bar{q}\,\hat{m}.$

$$= \int -\bar{q}\,\hat{m}\,\gamma_5\,\tau^a\, q$$

(1) + (2)

$$\rightarrow -\int \bar{q}\,\gamma_5\,\{\tau^a, \hat{m}\}\, q$$

method 2

$$[Q, \hat{O}] = -i \frac{\delta \hat{O}}{\delta \alpha}$$

↳ study how $\hat{O}$ transform under the given symmetry

$$\hat{O} \rightarrow \int \bar{q} \, \hat{m} \, q$$

$$q \rightarrow e^{-i \gamma_5 \tau^a \alpha^a_{AV}} q$$

$$\Delta \hat{O} \rightarrow \alpha^a_{AV} \int \bar{q} \left[(-i \gamma_5 \tau^a) \hat{m} + \hat{m} (-i \gamma_5 \tau^a) \right] q$$

$$[Q, \hat{O}] = - \int \bar{q} \, \gamma_5 \{ \tau^a, \hat{m} \} q$$

//

method 2 works well!

In fact

$$\left[Q^a_{AV}, \left[Q^a_{AV}, H \right] \right]$$

$$\longrightarrow -i \frac{d}{d\alpha} \left(-i \int \bar{q} \gamma_5 \{ \tau^a, \hat{m} \} q \right)$$

$$= (-i)(-i)(-1) \int \bar{q} \{ \tau^a, \{ \tau^a, \hat{m} \} \} q$$

$$= \int \bar{q} \{ \tau^a, \{ \tau^a, \hat{m} \} \} q \quad //$$

---

if $m_u = m_d$

$$\hat{m} \longrightarrow m \, \mathbb{I}_2.$$

no sum.

$$\{ \tau^a, \{ \tau^a, \hat{m} \} \} \longrightarrow 2 \{ \tau^a, \tau^a \} m.$$

$$= 4 \, \mathbb{I}_2 . \, m.$$

$$m_\eta^2 = \frac{-1}{f_\eta^2} \langle [Q^a_{AV}, [Q^a_{AV}, H]] \rangle$$

$$\longrightarrow \frac{-2m}{f_\eta^2} \langle \bar{u} u \rangle$$

$$= \langle \bar{d} d \rangle .$$