Hadrons Fall2023

HW2 due 01/12/2023

(+2 points for handing in on time)

Q1 Schwinger proper time regularization.

- Show that

$$\int_{-\infty}^{\infty} dx \, \frac{1}{x^2 + a^2} = (2\pi) \, \frac{1}{2a}.$$

- Make sense of the following identities:

$$\mathcal{A}^{-1} = \int_0^\infty dt \, e^{-t\mathcal{A}}$$
$$\ln \mathcal{A} = -\int_0^\infty dt \, \frac{1}{t} \left(e^{-t\mathcal{A}} - e^{-t\mathcal{I}} \right).$$

- With $\mathcal{A} \to p_4^2 + \omega^2$, show (again!) that

$$\int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \, \frac{1}{p_4^2 + \omega^2} = \frac{1}{2\sqrt{\omega^2}}.$$

Q2 Self-energy (part I).

- Explain why

$$\mathrm{Im} \left(\ln(-1 \pm i\delta) \right) \to \pm\pi$$

  where δ is a small (positive) number. Select any numerical method and verify these results.

- A QFT computation of the self energy Σ_R of a resonance (decaying to two daughters of equal mass m) gives

$$\Sigma_R(s) = 2 \times \frac{g^2}{16\pi^2} \int_0^1 dx \ln \left(m^2 - x(1-x)s - i\delta \right).$$

  (The factor of 2 is the symmetry factor for identical particles.) Show that the self energy develops an imaginary part when the COM energy is above the threshold:

$$\mathrm{Im}\Sigma_R(s) = -2 \times \frac{1}{2} \, g^2 \, \phi_2(s) \, \theta(\sqrt{s} - 2m)$$

  where $\phi_2(s)$ is the 2-body phase space. (HW01 Q4).

Q3 Recursive formula for N-body phase spaces.

The N-body Lorentz Invariant phase space (LISP) is defined as

$$\phi_N(s = P^2) = \int \frac{d^3p_1}{(2\pi)^3} \frac{1}{2E_1} \frac{d^3p_2}{(2\pi)^3} \frac{1}{2E_2} \cdots \frac{d^3p_N}{(2\pi)^3} \frac{1}{2E_N} \times$$
$$(2\pi)^4 \, \delta^4(P - \sum_i p_i).$$

Note that $E_j = \sqrt{p_j^2 + m_j^2}$.

- Show that the 3-body phase space satisfies the recursion formula

$$\phi_3(s, m_1^2, m_2^2, m_3^2) = \int_{s'_-}^{s'_+} \frac{ds'}{2\pi} \, \phi_2(s, s', m_3^2) \; \phi_2(s', m_1^2, m_2^2).$$

Work out the limits: $s'_\pm$.

Recall the 2-body phase space $\phi_2(s, m_1^2, m_2^2)$ is given by

$$\phi_2(s, m_1^2, m_2^2) = \frac{q(s)}{4\pi\sqrt{s}}$$
$$q(s) = \frac{1}{2} \sqrt{s} \sqrt{1 - \frac{(m_1 + m_2)^2}{s}} \sqrt{1 - \frac{(m_1 - m_2)^2}{s}}.$$

- Derive (again) the result for the massless case:

$$\phi_3(s) = \frac{s}{256\pi^3}.$$

Q4 Fourier transform.

a) Show that $\frac{e^{-mr}}{r}$ and $\frac{4\pi}{k^2+m^2}$ is a pair of (3D) Fourier transform.

b) Study the potential $V(r)$ obtained by transforming

$$V(k) = \frac{-8\pi}{(k^2 + \mu^2)^2}.$$

(μ is expected to be small.) Plot the potential $V(r)$ obtained. How the shape changes when we alter μ?

Q5 Gauge Transformation.

Consider the QED Lagrangian:

$$\mathcal{L}(x) = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m - e\gamma^\mu A_\mu(x)\right)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x),$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

a) Derive the equations of motion for ψ and A_μ fields.

b) Under a gauge transformation

$$\psi(x) \to e^{-i\alpha(x)}\psi(x)$$

with $\alpha(x)$ being a scalar field, how should $A_\mu(x)$ transform in order to preserve the Lagrangian? I.e.

$$\mathcal{L} \to \mathcal{L}.$$

c) Work out the Nother's current for the gauge transformation and show that it is conserved.