$$W_\omega^\Lambda = \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi} \ln(p_4^2+\omega^2)$$

$$\rightarrow \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi} \; -\int_{1/\Lambda^2}^{\infty} dt \, \frac{1}{t} e^{-t(p_4^2+\omega^2)} + \cdots$$

$$= -\int_{1/\Lambda^2}^{\infty} dt \; \frac{1}{t} \; \frac{1}{2\pi}\sqrt{\frac{\pi}{t}} \; e^{-t\omega^2} + \cdots$$

$$= \int_{1/\Lambda^2}^{\infty} \underline{dt \; \frac{-1}{t^{3/2}}} \; \frac{1}{2\sqrt{\pi}} \; e^{-t\omega^2} + \cdots$$

$2\,d(t^{-\frac{1}{2}})$

quite IR - enhanced.

$\Rightarrow$ inte. by parts

$-\frac{\Lambda}{\sqrt{\pi}}$

$$\rightarrow \left. 2\, t^{-\frac{1}{2}} \frac{1}{2\sqrt{\pi}} e^{-t\omega^2} \right|_{1/\Lambda^2}^{\infty}$$

$$+ \int_{1/\Lambda^2}^{\infty} \underline{dt \; \frac{\omega^2}{2\sqrt{\pi}} \; \frac{2}{\sqrt{t}} \; e^{-t\omega^2}}$$

$\omega + O(\frac{1}{\Lambda})$

this integral is finite even if $\Lambda \to \infty$

$$W_\omega^{\Lambda} \to \frac{-\Lambda}{\sqrt{\pi}} + \omega + \cdots$$

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in HW 4 Q3

you know that

$$W_\omega^{3-d} = \omega$$

& HW 4 Q4

$$\ln Z_F = \mathrm{tr} \ln (i\partial\!\!\!/ - m)$$

$$\longrightarrow \frac{1}{2} \mathrm{tr} \ln (\partial^2 + m^2)$$

$$= \frac{1}{2} V_4 \int \frac{d^4p}{(2\pi)^4} \ln\left[p_4^2 + (\vec{p}^{\,2} + m^2)\right] \overset{4}{\overset{\uparrow}{\mathrm{tr}_D I}}$$

$$= 2 V_4 \int \frac{d^3p}{(2\pi)^3} \underbrace{\frac{dp_4}{2\pi} \cdots \ln (p_4^2 + \varepsilon_p^2)}_{\varepsilon_{\vec{p}}}$$

$$\longrightarrow \underset{\beta V}{\underset{\updownarrow}{V_4}} \int \frac{d^3p}{(2\pi)^3} \, 2\sqrt{\vec{p}^{\,2} + m^2}$$

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