11. 2025

$A \longrightarrow 1 + 2 + 3.$ ①

$$\Phi_3(s) = \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3p_3}{(2\pi)^3} \frac{1}{8\varepsilon_1\varepsilon_2\varepsilon_3} \times$$

$$(2\pi)^4 \delta(\sqrt{s} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3) \; \delta^3(\vec{0} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

$$\longrightarrow \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{1}{8\varepsilon_1\varepsilon_2\varepsilon_3} \, 2\pi \, \delta(\sqrt{s} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3).$$

w. $\vec{p}_3 = -\vec{p}_1 - \vec{p}_2$

$$= \frac{1}{8\pi^3} \frac{1}{8\pi^3} \, 4\pi \, 2\pi \, \frac{1}{8} \, 2\pi \times$$

$$\int dp_1 \, dp_2 \, dz_{12} \, \frac{p_1^2 p_2^2}{\varepsilon_1\varepsilon_2\varepsilon_3} \, \delta(\sqrt{s} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3).$$

$$p_3 = \sqrt{p_1^2 + p_2^2 + 2p_1 p_2 z_{12}}$$

$$\varepsilon_3 = \sqrt{p_3^2 + m_3^2}$$

$$\longrightarrow \frac{1}{32\pi^3} \int_R d\varepsilon_1 \, d\varepsilon_2$$

where $\delta(\sqrt{s} - \varepsilon_1 - \varepsilon_2 - \varepsilon_3) = \dfrac{\delta(z_{12} - \bar{z})}{\dfrac{p_1 p_2}{\varepsilon_3}}$

w

$p_i \, dp_i = \varepsilon_i \, d\varepsilon_i$

the challenge is to figure out the range of integration

for ε_1 : max $\rightarrow$ $$\frac{s - m_1^2 + (m_2+m_3)^2}{2\sqrt{s}}.$$

$$2 \leftarrow \overset{\cdot}{1} \rightarrow 3 \qquad 1 \leftarrow \cdot \rightrightarrows {}^{2}_{3}.$$

Now for a specific ε_1 what is the range of ε_2 ?

$\delta(z_{12} - \bar{z})$ is effective if

$|\bar{z}| \leq 1$

$\bar{z}$ is obtained from

$$\sqrt{s} - \varepsilon_1 - \varepsilon_2 - \sqrt{p_1^2 + p_2^2 + 2p_1 p_2 \bar{z} + m_3^2} = 0$$

$|\bar{z}| \leq 1 \rightarrow$ range of ε_2

massless case

$$\sqrt{s} - p_1 - p_2 - \sqrt{p_1^2 + p_2^2 + 2 p_1 p_2 \bar{z}} = 0$$

$$\frac{\left[\sqrt{s} - (p_1 + p_2)\right]^2 - (p_1^2 + p_2^2)}{2 p_1 p_2} = \bar{z} \in (-1, 1)$$

$$-1 \leq \frac{s - 2\sqrt{s}(p_1 + p_2) + 2 p_1 p_2}{2 p_1 p_2} \leq 1$$

$\longrightarrow$ 1 $\quad s - 2\sqrt{s}(p_1 + p_2) \leq 0$

$$p_2 \geq \frac{1}{2}\sqrt{s} - p_1$$

$\longrightarrow$ a lower bound

$\longrightarrow$ 2 $\quad s - 2\sqrt{s}(p_1 + p_2) + 4 p_1 p_2 \geq 0$

$$p_2 \leq \frac{s - 2\sqrt{s}\, p_1}{2\sqrt{s} - 4 p_1} = \frac{\sqrt{s}}{2}$$

$$p_1 \leq \frac{\sqrt{s}}{2}$$

massless case

$$\frac{q}{s} \rightarrow \int_0^{\sqrt{s}/2} dp_1 \int_{\frac{1}{2}\sqrt{s}-p_1}^{\frac{1}{2}\sqrt{s}} dp_2 \; \frac{1}{32\pi^3}$$

$$= \frac{1}{32\pi^3} \; \frac{1}{2} p_1^2 \Big|_0^{\sqrt{s}/2}$$

$$= \frac{1}{256\pi^3} s$$

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